Online Appendix to Monopsony Power and Firm Organization

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A Appendix: Additional Figures and Tables

Table A.1: Classification of Occupations

Level	Tasks	Skills
Top Management	Definition of the firm general policy or consulting on the organization of the firm; strategic planning; creation or adaptation of technical, scientific and administrative methods or processes	Knowledge of management and coordination of firms fundamental activities; knowledge of management and coordination of the fundamental activities in the field to which the individual is assigned and that requires the study and research of high responsibility and technical level problems
Middle Management	Organization and adaptation of the guidelines established by the superiors and directly linked with the executive work	Technical and professional qualifications directed to executive, research, and management work
Supervisors	Orientation of teams, as directed by the superiors, but requiring the knowledge of action processes	Complete professional qualification with a specialization
Higher-skilled Professionals	Tasks requiring a high technical value and defined in general terms by the superiors	Complete professional qualification with a specialization adding to theoretical and applied knowledge
Skilled Professionals	Complex or delicate tasks, usually not repetitive, and defined by the superiors	Complete professional qualification implying theoretical and applied knowledge
Semi-skilled Professionals	Well defined tasks, mainly manual or mechanical (no intellectual work) with low complexity, usually routine and sometimes repetitive	Professional qualification in a limited field or practical and elementary professional knowledge
Non-skilled Professionals	Simple tasks and totally determined	Practical knowledge and easily acquired in a short time

Sources: (i) *Decreto-Lei nº. 121/78 de 2 de Junho, Ministério do Trabalho*, (ii) Caliendo et al. (2020).

Table A.2: Share of Occupations

Level	**Share (%)**	**Share Hierarchy (%)**	**Mean Wage**
Managers	19.2	100	1,696
Top Management	8	41.8	2,108
Middle Management	6	31.2	1,481
Supervisors and Team Leaders	5.2	27	1,308
Workers	80.8	100	717
Higher-skilled Professionals	8	9.9	1,192
Skilled Professionals	40.1	49.6	729
Semi-skilled Professionals	21.6	26.8	599
Non-skilled Professionals	11.1	13.7	562

Source: Elaboration based on QP.

Table A.3: Mobility and Sample Characteristics

	(1) Production Workers	(2) Managers
	Mean	Mean
Share Age≤25	0.11	0.04
Share Age≤30	0.25	0.17
Share Temporary	0.31	0.16
Share College	0.07	0.55
Share Change Establishment	0.10	0.08
Share Change Municipality	0.07	0.06
Share Change NUTS-3 Region	0.03	0.02
Share Change Sector	0.06	0.05
Observations	11,286,635	2,690,239

Source: Elaboration based on QP.

Note: All statistics are significantly different at standard statistical levels. We omit standard errors to ease readability.

Table A.4: Occupation and Migration across Municipalities

	(1)	(2)	(3)	(4)	(5)
Manager	-0.012***	-0.008***	-0.012***	-0.002***	-0.003***
	(0.0003)	(0.0003)	(0.0003)	(0.0003)	(0.0003)
AME/Baseline	-17.1%	-11.4%	-17.1%	-5.7%	-4.3%
Year FE	No	Yes	Yes	Yes	Yes
Sex	No	Yes	Yes	Yes	Yes
Age	No	Yes	Yes	Yes	Yes
Education	No	No	Yes	Yes	Yes
Temporary	No	No	No	Yes	Yes
Industry	No	No	No	No	Yes
N	6,628,978	6,628,978	6,615,462	6,572,412	6,572,412
Baseline	0.07	0.07	0.07	0.07	0.07

Source: Elaboration based on QP.

Note: The table reports the marginal effects from a Probit regression of inter-municipality migration on a manager dummy. In addition, to better interpret the results, we report the marginal effect relative to the baseline probability of migration of production workers. Each regression column differs in terms of the vector of controls. The sample period ranges from 2010 to 2016. Standard errors are reported in parentheses.

Table A.5: Occupation and Sectoral Mobility

	(1)	(2)	(3)	(4)	(5)
Manager	-0.012***	-0.008***	-0.014***	-0.005***	-0.006***
	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)
AME/Baseline	-20%	-13.3%	-23.3%	-8.3%	-10%
Year FE	No	Yes	Yes	Yes	Yes
Sex	No	Yes	Yes	Yes	Yes
Age	No	Yes	Yes	Yes	Yes
Education	No	No	Yes	Yes	Yes
Temporary	No	No	No	Yes	Yes
Industry	No	No	No	No	Yes
N	9,825,202	9,825,202	9,805,652	9,743,686	9,743,686
Baseline	0.06	0.06	0.06	0.06	0.06

Source: Elaboration based on QP.

Note: The table reports the marginal effects from a Probit regression of sectoral mobility (2-Digit) on a manager dummy. In addition, for a better interpretation of the results, we report the marginal effect relative to the baseline probability of sectoral mobility of production workers. Each regression column differs in terms of the vector of controls. The sample period ranges from 2010 to 2016. Standard errors are reported in parentheses.

Table A.6: Distribution of Number of Establishments across Local Markets (M_j)

	Mean	**P25**	**P50**	**P75**	**P95**
Markets of Managers					
Nº Establishments	120	3	11	39	156
Markets of Production Workers					
Nº Establishments	227	5	16	65	241

Source: Elaboration based on QP.

Note: The Table reports the (employment weighted) mean, the 25th, 50th, 75th, and 95th percentile of the number of establishments across local labor markets by occupation.

Figure A.1: Unbinding Minimum Wage in Partial Equilibrium Analysis

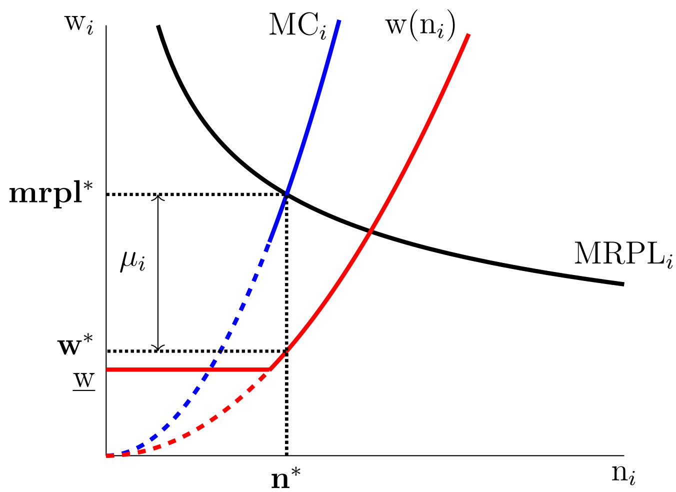


Figure A.2: Binding Minimum Wage on the Labor Supply Curve

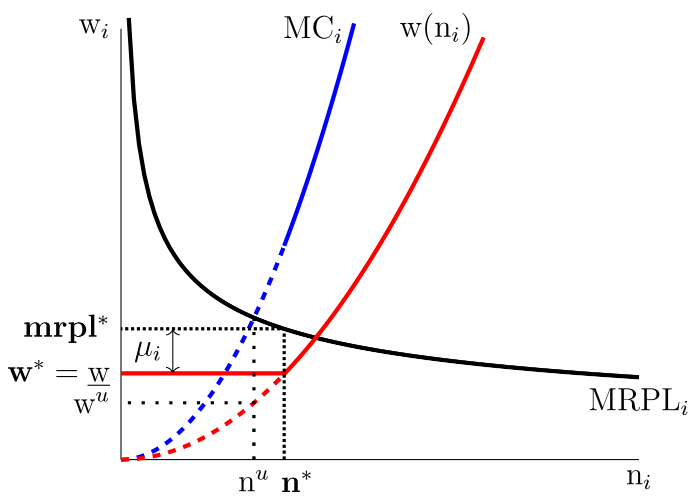


Figure A.3: Binding Minimum Wage off the Labor Supply Curve

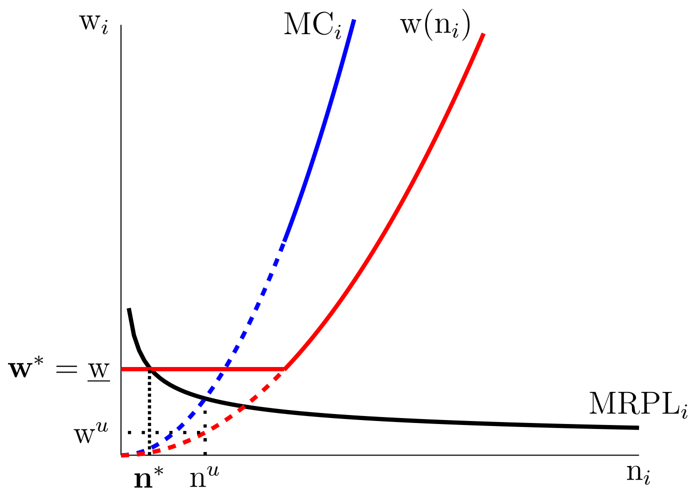
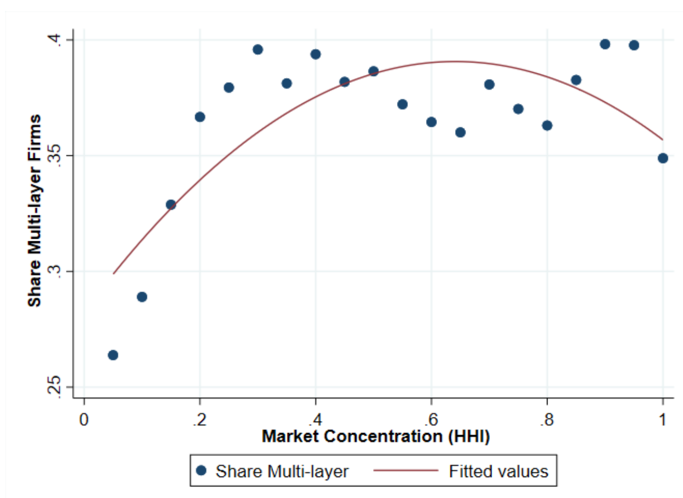


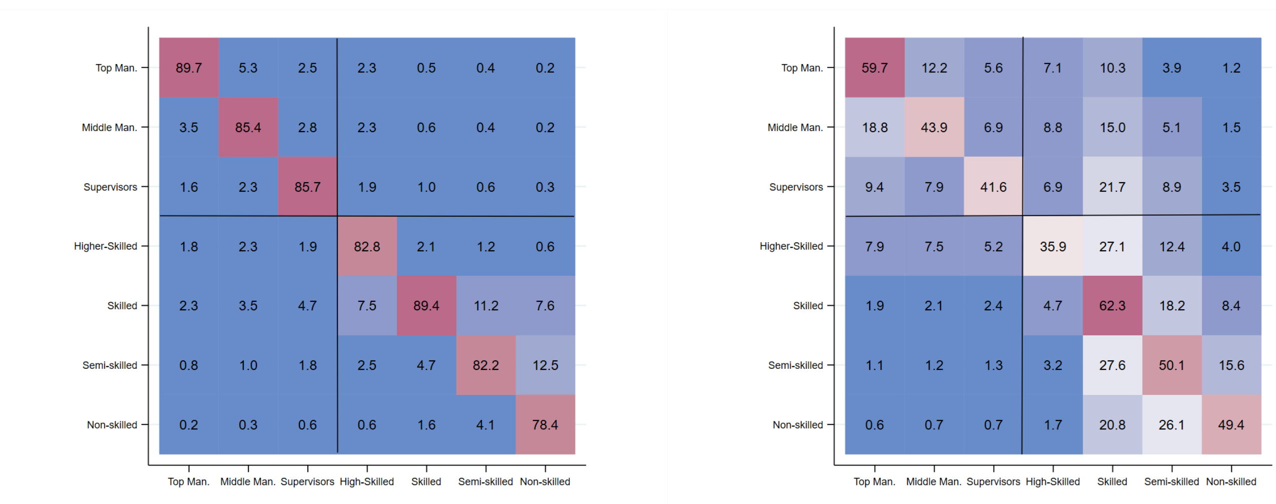
Figure A.4: Market Concentration and Multi-layer Firms: Unweighted



Source: Elaboration based on QP.

Note: The Figure plots the average share of multi-layer firms across local labor markets that differ in the level of HHI. In particular, we compute the share of multi-layer firms and the HHI for each local labor market of production workers. We split the distribution of the HHI into 20 cells of length 0.05. In each cell, we take the unweighted mean of the share of multi-layer firms across markets.

Figure A.5: Transition Probabilities



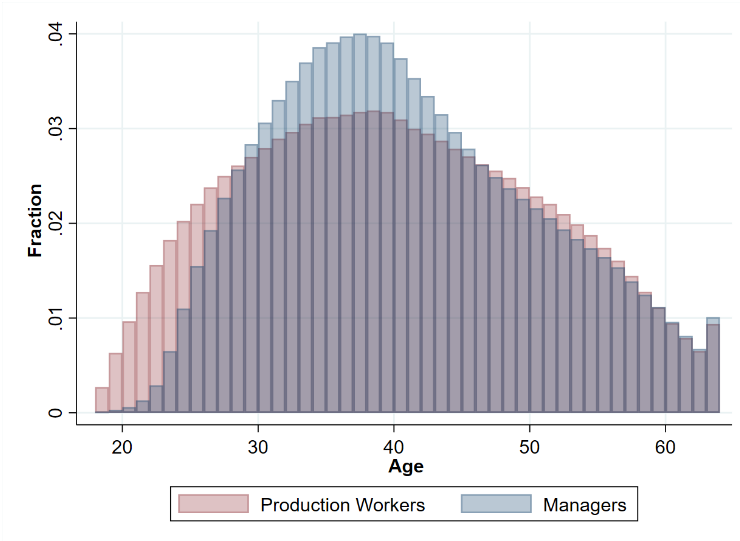
(a) Unconditional

(b) Conditional

Source: Elaboration based on QP.

Note: The Figures display the transition probabilities of changing sub-occupation. The vertical axis represents the sub-occupation before the transition, and the horizontal axis the sub-occupation afterward. The left panel shows the unconditional transition probability, whereas the right panel shows the transition probability conditional on changing firms. The black lines delimit the quadrants of moving across or within the two broad occupation categories (managers and production workers), where the top left and right bottom quadrants represent within-occupation transitions.

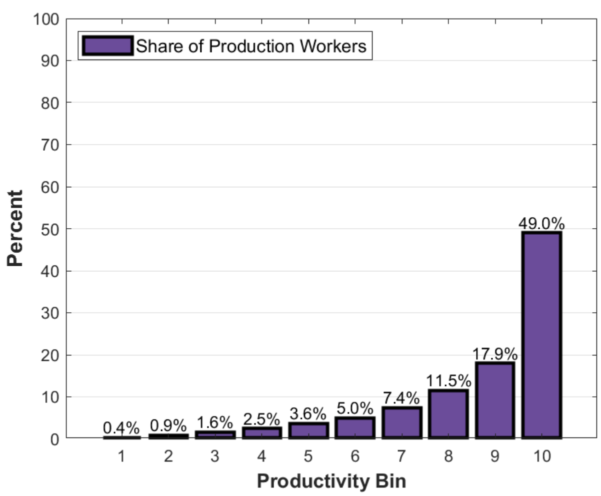
Figure A.6: Age Distribution across Occupations



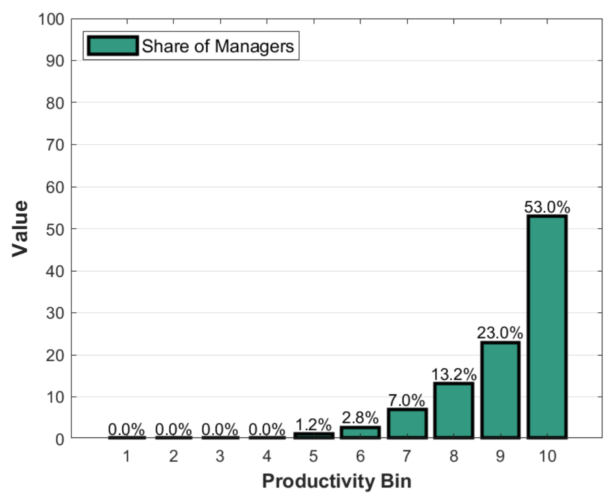
Source: Elaboration based on QP.

Note: The Figure displays the age distribution across occupations.

Figure A.7: Employment Share in the Benchmark Economy



(a) Production Workers

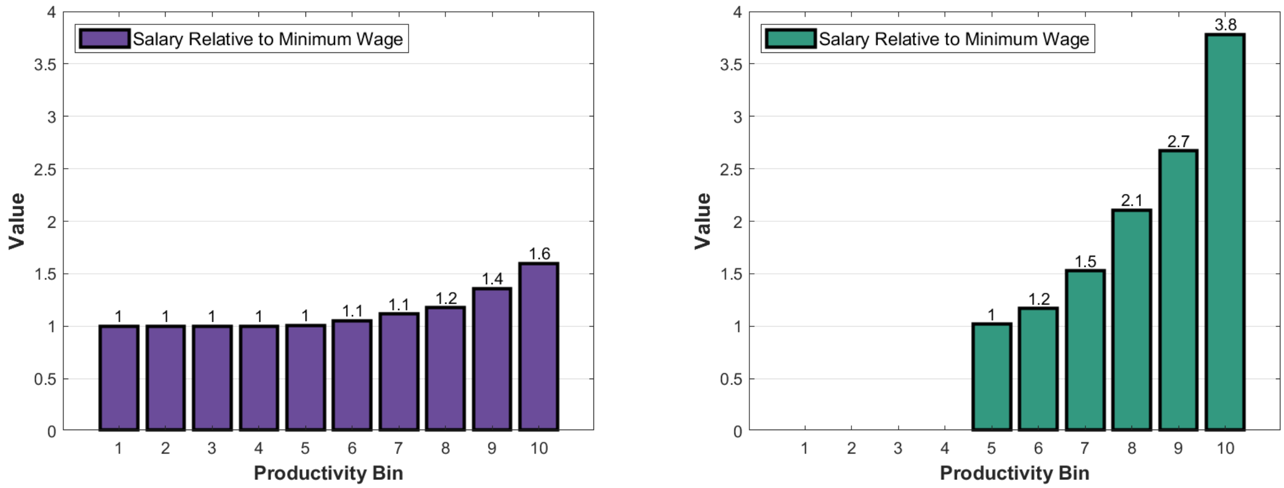


(b) Managers

Source: Simulations from the model.

Note: The Figure plots the employment share of production workers (left) and managers (right) across firms in the benchmark economy. We classify firms into ten bins according to their productivity, where a higher bin implies higher productivity.

Figure A.8: Mean Wages in the Benchmark Economy



(a) Production Workers

(b) Managers

Source: Simulations from the model.

Note: The Figure plots the employment-weighted mean wage of production workers (left) and managers (right) across firms in the benchmark economy. We classify firms into ten bins according to their productivity, where a higher bin implies higher productivity.

B Appendix: Data

This section provides a detailed description of the data, the occupation and the market definition, as well as the methodology to measure market concentration using the Herfindahl-Hirschman Index (HHI).

B.1 Quadros de Pessoal

Our primary data source is *Quadros de Pessoal* (QP), an annual census of private sector employees conducted by the Portuguese Ministry of Employment. This census provides matched employer-employee data on all firms based in Portugal with at least one worker, except those related to public administration and non-market services. The database incorporates unique time-invariant identifiers for each firm, establishment, and worker entering the report, which allows tracking them over time. Our sample covers the period from 2010 to 2016 for all results except for estimating the firm-substitutability parameters, which covers 2002 to 2016 because we require more observations.

The worker-level data contains information on each firm's employees as of the October reference week. The variables include age, occupation, monthly earnings, and hours worked. At the firm level, we have information on the industry, the headquarters location, and all its establishments.

Regarding the sample selection, we exclude workers younger than 18 or older than 64, those working outside of continental Portugal, and those working in agriculture, forestry, fishing, or mining industries. We also exclude apprentices, workers with missing information on earnings or occupation, and workers with misreported identifiers. Most workers with missing earnings include unpaid family members and owners of the firm. In addition, workers with misreported identifiers (e.g., duplicated) account for about 2% of the sample. Finally, we drop chief executive officers because their market is not local, which is a core feature of the theory in this paper. This selection results in 3,243,966 workers and 12,073,646 worker-year observations.

B.2 Market Definition

We classify labor markets based on three observable characteristics of the job: geography, industry, and occupation. This classification stems from the fact that workers are more attached to their current labor market because of imperfect geographical mobility and imperfect substitutability of skills across jobs and sectors (Neal, 1995; Kambourov and Manovskii, 2009; Sullivan, 2010; Monte et al., 2018). In particular, we define two broad occupations, i.e., managers and production workers, and define a local labor market for each occupation as the intersection of the geography (Municipality) and industry (2-digit NACE). This selection results in 13,832 and 11,677 local labor markets for workers and managers, respectively.

We use the municipality or *concelho* administrative division as the benchmark geographic unit, which splits the country into 278 areas of an average of 320 square kilometers. We use the 2-digit NACE classification of industries as a baseline measure. This includes 78 different economic sectors such as *Manufacture of food products* or *Accommodation and food service activities*. Given that our model does not distinguish between across-industry and across-region mobility, we use these baseline definitions because worker transitions are similar in both cases. In particular, the unconditional across-municipality and across-industry annual transition probabilities are 9.8 percent.

Regarding the occupational definition, the Portuguese law obliges firms to assign their workers to an occupational category based on tasks performed and skills required so that each category considers the level of the worker within the firm's hierarchy in terms of increasing responsibility and task complexity. We follow a hierarchical classification similar to Caliendo et al. (2020). In particular, we partition professional categories into two layers. We assign top executives, intermediary executives, supervisors, and team leaders to the management layer. In addition, we assign higher-skilled professionals, skilled professionals, skilled professionals, semi-skilled professionals, and non-skilled professionals to the bottom layer. To distinguish between managers and other occupations, the critical difference is that managers are responsible for the organizational policies of the firm and their adaptation, which require a high degree of qualification in terms of direction, guidance, and coordination of the firm

Table B.1: Summary Statistics at the Establishment Level

	Mean	P10	P25	P50	P75	P90
Production Workers						
Monthly Wage	718	518	588	756	1,082	2,159
Managers						
Monthly Wage	1,698	937	1,346	2,059	3,065	6,441
Span of Control	8	1	3	8	17	70

Source: Elaboration based on Quadros de Pessoal.

Note: The Table reports the mean, 10th, 25th, 50th, 75th, and 90th percentile of the individual distribution of wages for managers and non-managers. Wages are base wages (excluding supplementary payments) expressed in full-time equivalent units. In addition, it reports the same distributional moments for the span of control, which we define as the ratio of non-managers to managers within an establishment.

fundamental activities.[1]

B.3 Summary Statistics

Our classification of occupations implies that 19 percent of workers are managers, while the remaining 81 percent are production workers. Table B.1 reports summary statistics of the wage distribution for each occupation. Along the distribution, managers earn higher wages than production workers, and this gap particularly widens for high-paid workers. Managers earn around twice as much in the bottom quartile as production workers. In the top quartile, managers earn nearly three times as much as production workers.

This wage gap arises even though about two-thirds of managers are not top executives but supervisors, team leaders, or intermediary executives (see Table A.2). We measure the number of workers a manager has under his charge (the span of control) with the ratio of non-managers to managers in each establishment. In half of the establishments, managers have a span of control lower than eight workers, and only one-fourth of establishments have managers with a span of control greater than seventeen workers. These results highlight that most establishments assign a small span of control to their managers.

[1]See Table A.1 for further information about the categories of the occupational classification, which is based on *Decreto-Lei nº. 121/78 de 2 de Junho, Ministério do Trabalho.*

To summarize, we find substantial wage dispersion between managers and production workers. The literature on income inequality mainly attributes wage differences between groups to productivity-enhancing forces such as skill-biased technologies (**??**), task-biased technologies (**?**), and trade specialization (**?**). In this paper, we explore an additional force behind the wage dispersion between these occupations: heterogeneity in market competition.

B.4 Measuring Market Payroll Concentration

Our baseline measure of market concentration is the HHI. Given the employment n_{ijo} and wage w_{ijo} level at firm i in a local labor market j for occupation o, we define the HHI in the market as:

$$\mathrm{HHI}_{jo} := \sum_{i=1}^{M_{jo}} s_{ijo}^2 = \frac{1}{M_{jo}} + \sum_{i=1}^{M_{jo}} \left(s_{ijo} - \frac{1}{M_{jo}} \right)^2, \tag{1}$$

$$s_{ijo} := \frac{w_{ijo} n_{ijo}}{\sum_{i=1}^{M_{jo}} w_{ijo} n_{ijo}}. \tag{2}$$

Here, M_{jo} is the number of establishments in market j that hire workers in occupation o, and s_{ijo} stands for the payroll share of the firm i. The HHI equals the average payroll market share weighted by the payroll share itself. The index ranges from $\frac{1}{M}$ to 1, where a low value reflects low concentration or many firms having similar payroll shares. Note that this index gives more weight to larger establishments, especially penalizing markets where a few firms have a large share of the market payroll. The rightmost equality of Equation (1) shows that the HHI has an economically meaningful decomposition into two concentration sources. The first element involves the *number of establishments* in each market. All else being constant, increasing the number of establishments lowers the average establishment size in the market. The second element entails the *dispersion level of payroll shares* across establishments relative to the case in which they hold identical shares. All else being constant, increasing the dispersion in payroll shares leads to greater payroll concentration.

C Appendix: Derivations

C.1 Labor Supply

Microfoundations of the Nested CES Labor Supply

Closely following Berger et al. (2022), we provide a microfoundation of the labor supply curves in Equation (7). In particular, we show that this specification arises from a model in which individuals have heterogeneous idiosyncratic preferences for firms.

Consider a unit measure of ex-ante identical individuals indexed by $l \in [0,1]$ and a finite set of J local labor markets each populated by M_j firms. Suppose that workers derive utility according to the following specification:

$$U_{lij} = \log w_{ij} - \log y_{lij} + \log B_j + \zeta_{lij},$$

where ζ_{lij} is the idiosyncratic amenity that agent l derives from working at firm ij, w_{ij} stands for the wage that the agent earns by working h_{lij} hours at firm ij, the parameter B_j represents an amenity from working in market j, and $y_{lij} = w_{ij}h_{lij}$ represents earnings. Given a parameter $\theta > 0$, the following utility function represents the same preferences:

$$\tilde{U}_{lij} = (1+\theta)U_{lij} = (1+\theta)\Big(\log w_{ij} - \log y_{lij} + \log B_j\Big) + \tilde{\zeta}_{lij},$$

where the idiosyncratic amenity $\tilde{\zeta}_{lij}$ is distributed according to a multi-variate Gumbel distribution:

$$F(\tilde{\zeta}_{i1}, \ldots, \tilde{\zeta}_{iJ}) = \exp\Bigg[-\sum_{j=1}^{J}\Bigg(\sum_{i=1}^{M_j} e^{-\frac{\tilde{\zeta}_{ij}}{\rho}}\Bigg)^{\rho}\Bigg] = \exp\Bigg[-\sum_{j=1}^{J}\Bigg(\sum_{i=1}^{M_j} e^{-(1+\eta)\zeta_{ij}}\Bigg)^{\frac{1+\theta}{1+\eta}}\Bigg],$$
$$\rho = \frac{1+\theta}{1+\eta}.$$

The parameter ρ is a function of the correlation between the idiosyncratic amenities within each market j. Since the joint distribution of idiosyncratic amenities is a Generalized Extreme Value (GEV) distribution, then the probability that a worker l chooses firm ij has a closed-form solution equal to:[2]

$$\pi_{ij} = \underbrace{\frac{w_{ij}^{1+\eta}}{\sum_{i\in j} w_{ij}^{1+\eta}}}_{\text{Prob. worker chooses } i|j} \cdot \underbrace{\frac{\left[\sum_{i\in j}(B_j w_{ij})^{1+\eta}\right]^{\frac{1+\theta}{1+\eta}}}{\sum_{j=1}^{J}\left[\sum_{i\in j}(B_j w_{ij})^{1+\eta}\right]^{\frac{1+\theta}{1+\eta}}}}_{\text{Prob. worker chooses } j}.$$

Using the previous equation, we derive the firm-level labor supply:

$$n_{ij} = \int_0^1 \pi_{ij} h_{lij} dF(y_l) = \frac{w_{ij}^{\eta}}{\sum_{i\in j} w_{ij}^{1+\eta}} \cdot \frac{\left[\sum_{i\in j}(B_j w_{ij})^{1+\eta}\right]^{\frac{1+\theta}{1+\eta}}}{\sum_{j=1}^{J}\left[\sum_{i\in j}(B_j w_{ij})^{1+\eta}\right]^{\frac{1+\theta}{1+\eta}}} \cdot \underbrace{\int_0^1 w_{ij} h_{lij} dF(y_l)}_{=Y},$$

$$\Rightarrow\ n_{ij} = \frac{w_{ij}^{\eta}}{\sum_{i\in j} w_{ij}^{1+\eta}} \cdot \frac{\left[\sum_{i\in j}(B_j w_{ij})^{1+\eta}\right]^{\frac{1+\theta}{1+\eta}}}{\sum_{j=1}^{J}\left[\sum_{i\in j}(B_j w_{ij})^{1+\eta}\right]^{\frac{1+\theta}{1+\eta}}} \cdot Y.$$

To derive the expression of Equation (7), we first define the following wage and employment indexes:

$$\mathbf{w}_j = \left[\sum_{i\in j} w_{ij}^{1+\eta}\right]^{\frac{1}{1+\eta}} \quad , \qquad \mathbf{n}_j = \left[\sum_{i\in j} n_{ij}^{\frac{1+\eta}{\eta}}\right]^{\frac{\eta}{1+\eta}},$$

$$\mathbf{W} = \left[\sum_{j=1}^{J} \left(B_j \mathbf{w}_j\right)^{1+\theta}\right]^{\frac{1}{1+\theta}} \quad , \quad \mathbf{N} = \left[\sum_{j=1}^{J} \left(\frac{\mathbf{n}_j}{B_j}\right)^{\frac{1+\theta}{\theta}}\right]^{\frac{\theta}{1+\theta}}.$$

[2] For a comprehensive guide to additive random utility and nested models, see Chapter 15 in **?**.

Using these definitions and the previous labor supply curve, we show that $\mathbf{w}_j \mathbf{n}_j = \sum_{i\in j} w_{ij} n_{ij}$:

$$
\begin{aligned}
\sum_{i\in j} w_{ij} n_{ij} &= \frac{\left[\sum_{i\in j}(B_j w_{ij})^{1+\eta}\right]^{\frac{1+\theta}{1+\eta}}}{\sum_{j=1}^{J}\left[\sum_{i\in j}(B_j w_{ij})^{1+\eta}\right]^{\frac{1+\theta}{1+\eta}}} \cdot Y, \\
&= \underbrace{\left[\sum_{i\in j} w_{ij}^{1+\eta}\right]^{\frac{1}{1+\eta}} \cdot \frac{\left[\sum_{i\in j} w_{ij}^{1+\eta}\right]^{\frac{\eta}{1+\eta}}}{\sum_{i\in j} w_{ij}^{1+\eta}}}_{=1} \cdot \frac{\left[\sum_{i\in j}(B_j w_{ij})^{1+\eta}\right]^{\frac{1+\theta}{1+\eta}}}{\sum_{j=1}^{J}\left[\sum_{i\in j}(B_j w_{ij})^{1+\eta}\right]^{\frac{1+\theta}{1+\eta}}} \cdot Y, \\
&= \left[\sum_{i\in j} w_{ij}^{1+\eta}\right]^{\frac{1}{1+\eta}} \cdot \left[\frac{\sum_{i\in j} w_{ij}^{1+\eta}}{\left[\sum_{i\in j} w_{ij}^{1+\eta}\right]^{\frac{1+\eta}{\eta}}} \cdot \frac{\left[\sum_{i\in j}(B_j w_{ij})^{1+\eta}\right]^{\frac{1+\theta}{\eta}}}{\left(\sum_{j=1}^{J}\left[\sum_{i\in j}(B_j w_{ij})^{1+\eta}\right]^{\frac{1+\theta}{1+\eta}}\right)^{\frac{1+\eta}{\eta}}} \cdot Y^{\frac{1+\eta}{\eta}}\right]^{\frac{\eta}{1+\eta}}, \\
&= \left[\sum_{i\in j} w_{ij}^{1+\eta}\right]^{\frac{1}{1+\eta}} \cdot \left[\sum_{i\in j}\left(\frac{w_{ij}^{\eta}}{\sum_{i\in j} w_{ij}^{1+\eta}} \cdot \frac{\left[\sum_{i\in j}(B_j w_{ij})^{1+\eta}\right]^{\frac{1+\theta}{1+\eta}}}{\sum_{j=1}^{J}\left[\sum_{i\in j}(B_j w_{ij})^{1+\eta}\right]^{\frac{1+\theta}{1+\eta}}} \cdot Y\right)^{\frac{1+\eta}{\eta}}\right]^{\frac{\eta}{1+\eta}}, \\
&= \mathbf{w}_j \mathbf{n}_j.
\end{aligned}
$$

Next, we define $\tilde{w}_{ij} := B_j w_{ij}$ and show that $Y := \sum_j \mathbf{w}_j \mathbf{n}_j = \mathbf{WN}$

$$
\begin{aligned}
\mathbf{WN} &= \left[\sum_{j=1}^{J}\left(B_j \mathbf{w}_j\right)^{1+\theta}\right]^{\frac{1}{1+\theta}} \cdot \left[\sum_{j=1}^{J}\left(\frac{\mathbf{n}_j}{B_j}\right)^{\frac{1+\theta}{\theta}}\right]^{\frac{\theta}{1+\theta}}, \\
&= \left[\sum_{j=1}^{J} B_j^{1+\theta} \underbrace{\left(\sum_{i\in j} w_{ij}^{1+\eta}\right)^{\frac{1+\theta}{1+\eta}}}_{=\mathbf{w}_j^{1+\theta}}\right]^{\frac{1}{1+\theta}} \cdot \left[\sum_{j=1}^{J}\left(\frac{1}{B_j} \cdot \underbrace{\frac{1}{\left[\sum_{i\in j} w_{ij}^{1+\eta}\right]^{\frac{1}{1+\eta}}} \cdot \frac{\left[\sum_{i\in j} \tilde{w}_{ij}^{1+\eta}\right]^{\frac{1+\theta}{1+\eta}}}{\sum_{j=1}^{J}\left[\sum_{i\in j} \tilde{w}_{ij}^{1+\eta}\right]^{\frac{1+\theta}{1+\eta}}} \cdot Y}_{=\mathbf{n}_j}\right)^{\frac{1+\theta}{\theta}}\right]^{\frac{\theta}{1+\theta}}, \\
&= \frac{\left[\sum_{j=1}^{J}\left(\sum_{i\in j} \tilde{w}_{ij}^{1+\eta}\right)^{\frac{1+\theta}{1+\eta}}\right]^{\frac{1}{1+\theta}} \cdot \left[\sum_{j=1}^{J}\left(\sum_{i\in j} \tilde{w}_{ij}^{1+\eta}\right)^{\frac{1+\theta}{1+\eta}}\right]^{\frac{\theta}{1+\theta}}}{\sum_{j=1}^{J}\left(\sum_{i\in j} \tilde{w}_{ij}^{1+\eta}\right)^{\frac{1+\theta}{1+\eta}}} \cdot Y, \\
&= Y := \sum_j \mathbf{w_j n_j}.
\end{aligned}
$$

Therefore, plugging the aforementioned two expressions into the firms' labor supply equation yields the final expression in Equation (7):

$$
\begin{aligned}
n_{ij} &= \frac{w_{ij}^{\eta}}{\sum_{i\in j} w_{ij}^{1+\eta}} \cdot \frac{\left[\sum_{i\in j}(B_j w_{ij})^{1+\eta}\right]^{\frac{1+\theta}{1+\eta}}}{\sum_{j=1}^{J}\left[\sum_{i\in j}(B_j w_{ij})^{1+\eta}\right]^{\frac{1+\theta}{1+\eta}}} \cdot \mathbf{WN}, \\
&= \frac{w_{ij}^{\eta}}{\mathbf{w}_j^{1+\eta}} \cdot \frac{B_j^{1+\theta}\mathbf{w}_j^{1+\theta}}{\sum_{j=1}^{J} B_j^{1+\theta}\mathbf{w}_j^{1+\theta}} \cdot \mathbf{WN}, \\
&= \left(\frac{w_{ij}}{\mathbf{w}_j}\right)^{\eta} \cdot \frac{B_j^{1+\theta}\mathbf{w}_j^{\theta}}{\mathbf{W}^{1+\theta}} \cdot \mathbf{WN}, \\
\Rightarrow n_{ij} &= B_j^{1+\theta} \cdot \left(\frac{w_{ij}}{\mathbf{w_j}}\right)^{\eta} \cdot \left(\frac{\mathbf{w}_j}{\mathbf{W}}\right)^{\theta} \cdot \mathbf{N}.
\end{aligned}
$$

To get the expression for the inverse labor supply curve, we first need to compute the market-level supply curve:

$$
\begin{aligned}
\mathbf{n}_j &= \left[\sum_{i\in j} n_{ij}^{\frac{1+\eta}{\eta}}\right]^{\frac{\eta}{1+\eta}}, \\
&= \left[\sum_{i\in j}\left(\left(\frac{w_{ij}}{\mathbf{w_j}}\right)^{\eta} \cdot \left(\frac{\mathbf{w}_j}{\mathbf{W}}\right)^{\theta} \cdot B_j^{1+\theta} \cdot \mathbf{N}\right)^{\frac{1+\eta}{\eta}}\right]^{\frac{\eta}{1+\eta}}, \\
&= \left[\sum_{i\in j} w_{ij}^{1+\eta}\right]^{\frac{\eta}{1+\eta}} \cdot \frac{\mathbf{w}_j^{\theta}}{\mathbf{w}_j^{\eta}\mathbf{W}^{\theta}} \cdot \mathbf{N}B_j^{1+\theta}, \\
\Rightarrow \mathbf{n}_j &= \left(\frac{\mathbf{w}_j}{\mathbf{W}}\right)^{\theta} \cdot \mathbf{N}B_j^{1+\theta}.
\end{aligned}
$$

Then, we rearrange the market-level and the firms' labor supply curves:

$$
\begin{aligned}
\mathbf{w}_j &= \left(\frac{\mathbf{n}_j}{\mathbf{N}}\right)^{\frac{1}{\theta}} \cdot \frac{\mathbf{W}}{B_j^{\frac{1+\theta}{\theta}}}, \\
w_{ij} &= \left(\frac{n_{ij}}{\mathbf{n}_j}\right)^{\frac{1}{\eta}} \cdot \mathbf{w}_j.
\end{aligned}
$$

Lastly, plugging the inverse market-level supply curve into the inverse labor supply curve yields the final expression in Equation (7):

$$
w_{ij} = \frac{1}{B_j^{\frac{1+\theta}{\theta}}} \cdot \left(\frac{n_{ij}}{\mathbf{n}_j}\right)^{\frac{1}{\eta}} \cdot \left(\frac{\mathbf{n}_j}{\mathbf{N}}\right)^{\frac{1}{\theta}} \cdot \mathbf{W}.
$$

Solving the Household Problem

This section solves the household problem of Section 3. Each household type $o \in \{w, m\}$ choose consumption $\{c_{ijo}\}$ and the amount of labor supply $\{n_{ijo}\}$ to each firm to maximize their utility, taking as given wages:

$$\mathcal{U}_o = \max_{n_{ijo},c_{ijo}} \quad \mathbf{C}_o \;-\; \phi_o \frac{\mathbf{N}_o^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}},$$

subject to the household's budget constraint:[3]

$$\mathbf{C}_o = \int_0^1 \sum_{i=1}^{M_j} w_{ijo} n_{ijo} \, dj,$$

where we define the aggregate consumption and labor supply indexes as

$$\mathbf{C}_o := \int_0^1 \sum_{i=1}^{M_j} c_{ijo} \, dj$$

$$\mathbf{N_o} := \left[\int_0^1 \left(\frac{\mathbf{n_{jo}}}{B_{jo}} \right)^{\frac{\theta_o+1}{\theta_o}} dj \right]^{\frac{\theta_o}{\theta_o+1}} \qquad \mathbf{n_{jo}} := \left[\sum_{i=1}^{M_j} n_{ijo}^{\frac{\eta_o+1}{\eta_o}} \right]^{\frac{\eta_o}{\eta_o+1}}, \quad \eta_o > \theta_o > 0.$$

The Lagrangian of this maximization problem is:

$$\mathcal{L}(\{c_{ijo}\},\{n_{ijo}\},\lambda) \;=\; U_o(\{c_{ijo}\},\{n_{ijo}\}) \;+\; \lambda\Big(\int_0^1 \sum_{i=1}^{M_j} w_{ijo} n_{ijo} \, dj - \mathbf{C}_o \Big).$$

The first-order conditions associated with this problem are:

$$\frac{\partial \mathcal{L}}{\partial c_{ijo}} = 0 \iff \lambda = \frac{\partial U_o}{\partial \mathbf{C}_o} \cdot \frac{\partial \mathbf{C}_o}{\partial c_{ijo}} \iff \lambda = 1 \quad \forall ijo, \tag{C.1}$$

$$\frac{\partial \mathcal{L}}{\partial n_{ijo}} = 0 \iff \lambda w_{ijo} = \frac{\partial U_o}{\partial \mathbf{N}_o} \cdot \frac{\partial \mathbf{N}_o}{\partial \mathbf{n}_{jo}} \cdot \frac{\partial \mathbf{n}_{jo}}{\partial n_{ijo}} \quad \forall ijo. \tag{C.2}$$

Substituting Equation (C.1) into Equation (C.2) implies:

$$w_{ijo} = -\frac{\partial U_o}{\partial \mathbf{N}_o} \cdot \frac{\partial \mathbf{N}_o}{\partial \mathbf{n}_{jo}} \cdot \frac{\partial \mathbf{n}_{jo}}{\partial n_{ijo}}, \tag{C.3}$$

where each component of the last equation is equal to:

$$-\frac{\partial U_o}{\partial \mathbf{N}_o} = \phi_o \mathbf{N}^{\frac{1}{\gamma}}, \tag{C.4}$$

$$\frac{\partial \mathbf{N}_o}{\partial \mathbf{n}_{jo}} = \left(\frac{\mathbf{n}_{jo}}{\mathbf{N}_o} \right)^{\frac{1}{\theta}} \cdot \frac{1}{B_j^{\frac{1+\theta}{\theta}}}, \tag{C.5}$$

$$\frac{\partial \mathbf{n}_{jo}}{\partial n_{ijo}} = \left(\frac{n_{ijo}}{\mathbf{n}_{jo}} \right)^{\frac{1}{\eta}}. \tag{C.6}$$

[3]For simplicity, we omit the non-negativity constraints associated with consumption and labor supply. In the unconstrained solution, we will observe that such values also satisfy the constrained solution because they are always greater than or equal to zero.

Therefore, plugging the previous expressions into Equation (C.3):

$$w_{ijo} = \Big(\frac{n_{ijo}}{\mathbf{n}_{jo}}\Big)^{\frac{1}{\eta}} \cdot \Big(\frac{\mathbf{n}_{jo}}{\mathbf{N}_o}\Big)^{\frac{1}{\theta}} \cdot \frac{1}{B_j^{\frac{1+\theta}{\theta}}} \cdot \Big(-\frac{\partial U_o}{\partial \mathbf{N}_o}\Big). \tag{C.7}$$

To get the final expression of the firms' labor supply curve, we need to show that under optimality the aggregate wage is equal to the marginal disutility of aggregate labor supply. First, we define the market and aggregate wage indexes as follows:

$$\mathbf{w}_{jo} = \Big[\sum_{i\in j} w_{ij}^{\eta}\Big]^{\frac{1}{1+\eta}}, \tag{C.8}$$

$$\mathbf{W}_o = \Big[\sum_{j=1}^{J} \big(B_j \mathbf{w}_j\big)^{1+\theta}\Big]^{\frac{1}{1+\theta}}. \tag{C.9}$$

Substituting Equation (C.7) into Equation (C.8) implies:

$$\mathbf{w}_{jo} = \Big(\frac{\mathbf{n}_{jo}}{\mathbf{N}_o}\Big)^{\frac{1}{\theta}} \cdot \frac{1}{B_j^{\frac{1+\theta}{\theta}}} \cdot \Big(-\frac{\partial U_o}{\partial \mathbf{N}_o}\Big). \tag{C.10}$$

Then, substituting the last equation into Equation (C.9) yields the desired result:

$$\mathbf{W}_o = -\frac{\partial U_o}{\partial \mathbf{N}_o}. \tag{C.11}$$

Moreover, we derive the expression for the aggregate labor supply disutility in Equation (6) by plugging Equation (C.4) into Equation (C.11) and rearranging:

$$\mathbf{N}_o = \Big(\frac{\mathbf{W}_o}{\phi_o}\Big)^{\gamma}.$$

Finally, we get the final expression for the firms' labor supply curve. Note that Equation (C.7) and Equation (C.11) imply the inverse firms' labor supply curve in Equation (7):

$$w_{ijo} = \frac{1}{B_j^{\frac{1+\theta}{\theta}}} \cdot \Big(\frac{n_{ijo}}{\mathbf{n}_{jo}}\Big)^{\frac{1}{\eta}} \cdot \Big(\frac{\mathbf{n}_{jo}}{\mathbf{N}_o}\Big)^{\frac{1}{\theta}} \cdot \mathbf{W}_o.$$

Moreover, substituting Equation (C.11) into Equation (C.10) and rearranging imply:

$$\mathbf{n}_j = \Big(\frac{\mathbf{w}_j}{\mathbf{W}}\Big)^{\theta} \cdot \mathbf{N} B_j^{1+\theta},$$

and substituting Equation (C.10) into Equation (C.7) and rearranging imply:

$$n_{ijo} = \Big(\frac{w_{ijo}}{\mathbf{w}_j}\Big)^{\eta} \cdot \mathbf{n}_j.$$

Hence, using the last two equations yields the expression for the firms' labor supply curve in Equation (7):

$$n_{ij} = B_j^{1+\theta} \cdot \big(\frac{w_{ij}}{\mathbf{w_j}}\big)^{\eta} \cdot \big(\frac{\mathbf{w}_j}{\mathbf{W}}\big)^{\theta} \cdot \mathbf{N}.$$

C.2 Labor Demand

This section solves the firm problem of Section (3) for single-layer firms. Since we solve the organizational problem of choosing the number of layers numerically, we proceed to analytically solve the profit maximization problem subsequent to adopting the organizational structure. Moreover, we omit the problem for multi-layer firms for illustrative purposes, as the additional complexity added by this problem compared to the problem of single-layer firms only involves an expanded set of scenarios to analyze.

When firms adopt a single-layer structure, they also choose the measure of production workers n_{ijw} to maximize profits, given the employment policies of their local competitors, n^*_{-ijw}. In particular, they solve:

$$\pi(z,1) \;=\; \max_{n_{ijw}} \quad y(z,1) \;-\; w_{ijw}\Big(n_{ijw}, n^*_{-ijw}, \mathbf{N}_w, \mathbf{W}_w\Big)n_{ijw},$$

subject to the inverse labor supply curve of production workers and minimum wages:

$$w_{ijw}\Big(n_{ijw}, n^*_{-ijw}, \mathbf{N}_w, \mathbf{W}_w\Big) = \left(\frac{1}{B_{jw}}\right)^{\frac{1+\theta_o}{\theta_o}} \left(\frac{n_{ijw}}{\mathbf{n}_{jw}(n_{ijw}, n^*_{-ijw})}\right)^{\frac{1}{\eta_w}} \left(\frac{\mathbf{n}_{jw}(n_{ijw}, n^*_{-ijw})}{\mathbf{N}_w}\right)^{\frac{1}{\theta_w}} \mathbf{W}_w,$$

$$\mathbf{n}_{jw}(n_{ijw}, n^*_{-ijw}) = \left[n_{ijw}^{\frac{1+\eta_w}{\eta_w}} + \sum_{k\neq i} n^{*}_{kjw}{}^{\frac{1+\eta_w}{\eta_w}}\right]^{\frac{\eta_w}{1+\eta_w}},$$

$$w_{ijw} \geq \underline{w}$$

The associated Lagrangian function is:

$$\mathcal{L}(n_{ijw}, \mu) \;=\; y(z,1) \;-\; w_{ijw}n_{ijw} \;+\; \nu \cdot \Big(w_{ijw} - \underline{w}\Big).$$

To ease notation, we omit that the following conditions hold when employment is optimal and that the firm internalizes an inverse labor supply that is a function of the labor supply of all competitors within the market. The system of Kuhn-Tucker conditions is given by:

$$\frac{\partial \mathcal{L}}{\partial n_{ijw}} = 0 \iff \frac{\partial y(z,1)}{\partial n_{ijw}} + \nu = \frac{\partial w_{ijw}}{\partial n_{ijw}} \cdot n_{ijw} + w_{ijw}, \tag{C.12}$$

$$\nu \cdot \Big(w_{ijw} - \underline{w}\Big) = 0, \tag{C.13}$$

$$\nu \geq 0, \tag{C.14}$$

$$w_{ijw} = \max\Bigg(\underline{w}\ ,\ \underbrace{\left(\frac{1}{B_{jw}}\right)^{\frac{1+\theta_o}{\theta_o}} \left(\frac{n_{ijw}}{\mathbf{n}_{jw}}\right)^{\frac{1}{\eta_w}} \left(\frac{\mathbf{n}_{jw}}{\mathbf{N_w}}\right)^{\frac{1}{\theta_w}} \mathbf{W}_w}_{=\tilde{w}_{ijw}\text{ ,i.e., unconstrained labor supply curve}}\Bigg), \tag{C.15}$$

$$\mathbf{n}_{jw} = \left[n_{ijw}^{\frac{1+\eta_w}{\eta_w}} + \sum_{k\neq i} n_{kjw}^{*\ \frac{1+\eta_w}{\eta_w}}\right]^{\frac{\eta_w}{1+\eta_w}}. \tag{C.16}$$

To solve the maximization problem, we break the problem into three different cases.

Case I: The minimum wage is not binding. Suppose the case when the minimum wage is not binding $w^*_{ijw} > \underline{w}$. Then, Equation (C.13) implies that $\nu = 0$, and Equation (C.12) is given by:

$$\begin{aligned}
\frac{\partial y(z,1)}{\partial n_{ijw}}\Big|_{n^*_{ijw}} &= n^*_{ijw} \cdot \frac{\partial w_{ijw}}{\partial n_{ijw}}\Big|_{n^*_{ijw}} + w^*_{ijw}, \\
\frac{\partial y(z,1)}{\partial n_{ijw}}\Big|_{n^*_{ijw}} &= \frac{w^*_{ijw}}{\varepsilon^*_{ijw}} + w^*_{ijw}, \\
\Rightarrow\ w^*_{ijw} &= \mu^*_{ijw} \cdot \frac{\partial y(z,1)}{\partial n_{ijw}}\Big|_{n^*_{ijw}}.
\end{aligned}$$

where ε_{ijw} is the structural elasticity of labor supply and μ_{ijw} represents the wage markdown:

$$\begin{aligned}
\varepsilon_{ijo} &= \left[\frac{\partial \text{log} w_{ijo}}{\partial \text{log} n_{ijo}}\right]^{-1}, \\
\mu_{ijw} &= \frac{\varepsilon_{ijw}}{\varepsilon_{ijw}+1} \in [0,1].
\end{aligned}$$

In Appendix C.3 we show that the structural elasticity of labor supply has a closed-form solution given by:

$$\varepsilon_{ijo}(s_{ijo}) = \left[\frac{1}{\eta_o} + \left(\frac{1}{\theta_o} - \frac{1}{\eta_o}\right)\frac{\partial \log \mathbf{n}_{jo}}{\partial \log n_{ijo}}\right]^{-1} = \left[\frac{1}{\eta_o} + \left(\frac{1}{\theta_o} - \frac{1}{\eta_o}\right) s_{ijo}\right]^{-1}, \tag{C.17}$$

where s_{ijo} is the payroll share of firm i in market j:

$$s_{ijo} := \frac{w_{ijo} n_{ijo}}{\sum_{i\in j} w_{ijo} n_{ijo}}. \tag{C.18}$$

Case II: The minimum wage is binding, and labor supply equals labor demand. Suppose the case when the minimum wage is binding $w^*_{ijw} = \underline{w}$ and labor supply equals labor demand, that is, Equation (C.15) satisfies:

$$w^*_{ijw} = \underline{w} \;\; = \;\; \left(\frac{1}{B_{jw}}\right)^{\frac{1+\theta_o}{\theta_o}} \left(\frac{n^*_{ijw}}{\mathbf{n}^*_{jw}}\right)^{\frac{1}{\eta_w}} \left(\frac{\mathbf{n}^*_{jw}}{\mathbf{N_w}}\right)^{\frac{1}{\theta_w}} \mathbf{W}_w \tag{C.19}$$

Then, the optimal level of employment is given by Equation (C.19). Moreover, the Lagrange multiplier associated with the inequality constraint may not be binding, i.e., $\nu \geq 0$. Thus, Equation (C.12) implies that the marginal revenue must be smaller or equal than the marginal cost. In contrast, the marginal revenue must be greater or equal than the minimum wage. We prove this by contradiction. Suppose $\underline{w} > \text{mrpl}(n^*_{ijw})$. Since the unconstrained labor supply curve is strictly increasing in labor, then Equation (C.15) implies that $w(n_{ijw}) = \underline{w} \;\;\; \forall n_{ijw} < n^*_{ijw}$. Thus, the marginal cost is also equal to the minimum wage within this employment range: $\text{mc}(n_{ijw}) = \underline{w} \;\;\; \forall n_{ijw} < n^*_{ijw}$. Moreover, since the marginal revenue of labor is strictly decreasing in labor units, then there exists a threshold $n'_{ijw} < n^*_{ijw}$ for which $\text{mrpl}(n'_{ijw}) = \underline{w}$ and $\text{mrpl}(n_{ijw}) < \underline{w} \;\;\; \forall n_{ijw} \in (n'_{ijw}, n^*_{ijw}]$. However, this implies that n'_{ijw} is feasible and more profitable than n^*_{ijw} because any unit between them yields negative profits, i.e., their marginal cost is higher than their marginal revenue, which contradicts that n^*_{ijw} is optimal. Hence, it must be the case that $\underline{w} \leq \text{mrpl}(n^*_{ijw})$.

Overall, the previous results imply that:

$$w^*_{ijw} \;=\; \underline{w}, \tag{C.20}$$

$$\left.\frac{\partial y(z,1)}{\partial n_{ijw}}\right|_{n^*_{ijw}} \;\leq\; \left.\frac{\partial w_{ijw}}{\partial n_{ijw}}\right|_{n^*_{ijw}} + \underline{w}, \tag{C.21}$$

$$\left.\frac{\partial y(z,1)}{\partial n_{ijw}}\right|_{n^*_{ijw}} \;\geq\; \underline{w}. \tag{C.22}$$

Here, the markdown does not have a closed-form solution but is given by:

$$\mu_{ijw} \;=\; \frac{\underline{w}}{\left.\frac{\partial y(z,1)}{\partial n_{ijw}}\right|_{n^*_{ijw}}} \in [0,1]. \tag{C.23}$$

Case III: The minimum wage is binding, and labor supply exceeds labor demand. Suppose the case when the minimum wage is binding $w^*_{ijw} = \underline{w}$ and labor supply excess labor demand, that is, Equation (C.15) satisfies:

$$w^*_{ijw} = \underline{w} \; > \; \left(\frac{1}{B_{jw}}\right)^{\frac{1+\theta_o}{\theta_o}} \left(\frac{n^*_{ijw}}{\mathbf{n}^*_{jw}}\right)^{\frac{1}{\eta_w}} \left(\frac{\mathbf{n}^*_{jw}}{\mathbf{N_w}}\right)^{\frac{1}{\theta_w}} \mathbf{W}_w. \tag{C.24}$$

Next, we note that the previous fact implies that the marginal cost of an additional hire is the minimum wage. Graphically, the wage function is flat in a neighborhood of the optimal labor choice n^*_{ijw}, as displayed in Figure A.3. To prove it mathematically, we also rely on the fact that the unconstrained labor supply curve is strictly increasing in labor. Thus, $\tilde{w}_{ijw}(\tilde{n}_{ijw}) = \underline{w}$ for $\tilde{n}_{ijw} > n^*_{ijw}$. Then, there exists $\bar{\varepsilon} > 0$ such that $\varepsilon \in [0, \bar{\varepsilon}]$ and $n'_{ijw} = n^*_{ijw} + \varepsilon < \tilde{n}_{ijw}$ for which $w^*_{ijw}(n'_{ijw}) = \underline{w}$. As a result, $\frac{\partial w_{ijw}}{\partial n_{ijw}} = 0$.

Moreover, this case also implies that $\nu = 0$. We prove this by contradiction. Suppose $\nu > 0$, then Equation (C.12) implies that $\frac{\partial y(z,1)}{\partial n_{ijw}}\Big|_{n^*_{ijw}} < \underline{w}$. Since we assume that the marginal productivity is strictly decreasing in labor, there exists $\varepsilon > 0$ such that $n''_{ijw} = n^*_{ijw} - \varepsilon$ and $\frac{\partial y(z,1)}{\partial n_{ijw}}\Big|_{n''_{ijw}} = \underline{w} > \frac{\partial y(z,1)}{\partial n_{ijw}}|_{n^*_{ijw}}$. Then, choosing n''_{ijw} and paying them $\underline{w}$ is feasible and more profitable because it raises revenue while keeping costs fixed. This is a contradiction because n^*_{ijw} is optimal. Hence, it must be $\nu = 0$.

Therefore, the aforementioned two results and Equation (C.12) imply that labor demand satisfies:

$$w^*_{ijw} \;=\; \underline{w} \;=\; \frac{\partial y(z,1)}{\partial n_{ijw}}\Big|_{n^*_{ijw}}, \qquad \mu_{ijw} = 1. \tag{C.25}$$

Where the markdown equals one by definition.

C.3 Structural Elasticity of Labor Supply

To show that the structural elasticity has a closed-form solution, consider the log transformation of the inverse labor supply curve in Equation (7):

$$\log\Big(w(n_{ijo}, n^*_{-ijo}, \mathbf{W}_o, \mathbf{N}_o)\Big) \;=\; \frac{1}{\eta_o}\log(n_{ijo}) + \left(\frac{1}{\theta_o} - \frac{1}{\eta_o}\right)\log(\mathbf{n}_{jo}) - \frac{1}{\theta_o}\log(\mathbf{N}_o) + \log(\mathbf{W}_o) - \frac{1+\theta_o}{\theta_o}\log(B_{jo}).$$

Thus,

$$\frac{\partial \log\Big(w(n_{ijo}, n^*_{-ijo}, \mathbf{W}_o, \mathbf{N}_o)\Big)}{\partial \log(n_{ijo})} \;=\; \frac{1}{\eta_o} \;-\; \left(\frac{1}{\theta_o} - \frac{1}{\eta_o}\right)\frac{\partial \log(\mathbf{n}_{jo})}{\partial \log(n_{ijo})}.$$

Note that the derivative of the economy-wide variables with respect to the firm's employment is zero because firms are atomistic with respect to the economy. Moreover, the definition of the market labor supply disutility index implies that:

$$\begin{aligned}
\frac{\partial \log(\mathbf{n}_{jo})}{\partial \log(n_{ijo})} &= \frac{\partial \mathbf{n}_{jo}}{\partial n_{ijo}} \cdot \frac{n_{ijo}}{\mathbf{n}_{jo}}, \\
&= n_{ijw}^{\frac{1+\eta_o}{\eta_o}} \cdot \mathbf{n}_{jo}^{-1} \cdot \left(\sum_{i \in j} n_{ijo}^{\frac{\eta_o+1}{\eta_o}} \right)^{-\frac{1}{\eta_o+1}}, \\
&= \left(\frac{n_{ijo}}{\mathbf{n}_{jo}} \right)^{\frac{1+\eta_o}{\eta_o}}.
\end{aligned}$$

Therefore,

$$\frac{\partial \log\Big(w(n_{ijo}, n^*_{-ijo}, \mathbf{W}_o, \mathbf{N}_o) \Big)}{\partial \log(n_{ijo})} = \frac{1}{\eta_o} - \left(\frac{1}{\theta_o} - \frac{1}{\eta_o} \right) \cdot \left(\frac{n_{ijo}}{\mathbf{n}_{jo}} \right)^{\frac{1+\eta_o}{\eta_o}}.$$

Next, we show that the last fraction is equal to the payroll share of the firm i in market j. In particular, defining the payroll share and substituting the inverse labor supply curve:

$$\begin{aligned}
s_{ijo} &= \frac{w_{ijo} n_{ijo}}{\sum_{i \in j} w_{ijo} n_{ijo}} = \frac{n_{ijo}^{\frac{1+\eta_o}{\eta_o}}}{\sum_{i \in j} n_{ijo}^{\frac{1+\eta_o}{\eta_o}}}, \\
&= \left(\frac{n_{ijo}}{\mathbf{n}_{jo}} \right)^{\frac{1+\eta_o}{\eta_o}}.
\end{aligned}$$

Hence, the structural labor supply elasticity is given by:

$$\varepsilon_{ijo} := \left[\frac{\partial \log w(n_{ijo}, n^*_{-ijo}, \mathbf{W}_o, \mathbf{N}_o)}{\partial \log n_{ijo}} \right]^{-1} = \left[\frac{1}{\eta_o} - \left(\frac{1}{\theta_o} - \frac{1}{\eta_o} \right) s_{ijo} \right]^{-1}.$$

We take the concept of "structural" from Berger et al. (2022). The motivation for this concept is twofold. First, it arises from a structural macroeconomic model with firm granularity and strategic interaction in labor demand. In these models, the structural elasticity is the welfare-relevant variable of the model because its distribution determines the distribution of wage markdowns. Second, it is useful to distinguish the concept from the more commonly estimated reduced-form labor supply elasticity. On the one hand, the structural elasticity is the labor supply elasticity faced by a firm that internalizes the employment responses of its competitors within the local labor market. It measures the percent change in the firm's labor supply due to an increase of one percent in the firm's wages, holding its competitors' employment constant. On the other hand, the reduced-form elasticity measures the percent change in the firm's labor supply due to an increase of one percent in the firm's wages. Thus, in our model, this variable includes the effect of the response of the firm's competitors on the firm's own wage. For example, when a firm receives an idiosyncratic positive shock and increases labor demand, Cournot competition implies that the firm's competitors best respond by decreasing labor demand, which also leads the shocked firm to best respond and increase its quantity of labor demand and so on.

D Appendix: Algorithm

The solution of the equilibrium consists of a fixed point in wages. We solve the problem for 20 different firm productivity grids and 1,000 markets. To ease the algorithm's interpretation, we first describe the equilibrium solution in the absence of minimum wages.

D.1 Algorithm with No Minimum Wages

The idea of the algorithm is that whenever a firm faces an excess of labor demand for one occupation, then we smoothly increase its wage. In contrast, we smoothly decrease it whenever it faces an excess of labor supply. Thus, the algorithm always converges as long as the labor supply curve firms face is strictly increasing and the marginal revenue is strictly decreasing in employment.

We initialize the algorithm by guessing a vector of wages $\left\{w_{ijo}^{(0)}\right\}_{\forall ijo}$. Consider iteration k:

1. **Compute labor supply**.
   Note that firms' wages $w_{ijo}^{(k)}$ are enough to get $\mathbf{w}_{jo}^{(k)}$, $\mathbf{W}_{o}^{(k)}$, and $\mathbf{N}_{o}^{(k)}$ from Equations (6) and (8). Then, we compute the labor supply to each firm $n_{ijo}^{s,(k)}$ by substituting the previous variables into Equation (7).

2. **Compute organizational choice.**
   Market clearing implies that labor supply is equal to labor demand in equilibrium. Thus, labor supply is equal to labor demand at equilibrium wages $n_{ijo}^{d,(k)} = n_{ijo}^{s,(k)}$. Then, we use wages $\left\{w_{ijo}^{(k)}\right\}_{\forall ijo}$ and labor demands $\left\{n_{ijo}^{d,(k)}\right\}_{\forall ijo}$ to compute the optimal

organizational choice $\ell^{(k)}$ for all firms using the same method that we describe in the previous subsection. Note that wages and employment change when we update the firm's optimal organizational structure, e.g., we set the wage and employment of managers to zero for single-layer firms: $w_{ijm}^{(k)} = 0$ and $n_{ijm}^{s,(k)} = n_{ijm}^{d,(k)} = 0$ if $\ell^{(k)} = 1$.

3. **Compute markdowns**.
   We compute the payroll market share $s_{ijo}^{(k)}$ using wages $w_{ijo}^{(k)}$ and labor demand $n_{ijo}^{d,(k)}$ of all firms in market j. Then, we compute the firm's structural elasticity $\varepsilon_{ijo}^{(k)}$ and markdown $\mu_{ijo}^{(k)}$ from Equations (12)-(13).

4. **Compute wages from labor demand FOCs.**
   For each occupation, we use the optimal organization $\ell^{(k)}$ and labor demand $n_{ijo}^{d,(k)}$ of each firm to compute its marginal revenue product of labor:

$$\text{mrpl}_{ijo}^{(k)} = \left.\frac{\partial y(z, \ell^{(k)})}{\partial n_{ijo}}\right|_{n_{ijo}^{d,(k)}}.$$

   Then, we update the occupation-specific wages of all firms as:

$$w_{ijo}^{\prime\,(k)} = \begin{cases} \mu_{ijo}^{(k)} \cdot \text{mrpl}_{ijo}^{(k)}, & \text{if } n_{ijo}^{d,(k)} > 0, \\ 0, & \text{if } n_{ijo}^{d,(k)} = 0. \end{cases}$$

5. **Iteration**.
   Iterate over (1) to (4) until convergence of wages. Whenever $\max\left\{\text{abs}(w_{ijo}^{(k)} - w_{ijo}^{\prime\,(k)})\right\} >$ tol, we update wages with the following criterion:

$$w_{ijo}^{(k+1)} = \rho w_{ijo}^{(k)} + (1-\rho) w_{ijo}^{\prime\,(k)} \quad \text{for} \quad \rho \in (0,1).$$

D.2 Computing the Optimal Organization Structure

Calculating the maximum profits a firm could earn by opting for the off-equilibrium organizational structure is the main challenge when determining a firm's optimal organizational structure. This is because computing off-equilibrium profits in each iteration would be overly computationally expensive. To address this issue, we proceed as follows.

We start by setting a slightly higher tolerance level than the main algorithm's. When the distance between the initial and predicted wages exceeds this tolerance, we assign the following off-equilibrium wages and employment to each firm:

- If the firm began the iteration as a multi-layer organization, we use its equilibrium wage and employment levels for production workers to calculate its profits as a single-layer organization.

- If the firm began the iteration as a single-layer organization, we use the equilibrium wage and employment levels for production workers and managers of its nearest competitor to compute the firm's profits as a multi-layer organization. When there exist multi-layer firms within the same market, the nearest competitor is the multi-layer firm in the market located in the closest productivity bin. If no multi-layer firms are within the same market, we assign economy-wide minimum wage and employment levels for both occupations. Finally, we utilize this information to compute the firms' profits on- and off-equilibrium for each organizational structure and solve the Problem (9).

Next, when the distance between the initial and predicted wages falls below this tolerance, it indicates that the algorithm is close to converging. In such cases, we compute the actual wage and employment levels for each firm's off-equilibrium organizational structure using a numerical solution. Specifically, we calculate off-equilibrium profits by numerically solving Problem (10) for multi-layer firms and Problem (11) for single-layer firms. With this information and the optimal profits for firms in equilibrium, we determine the optimal organizational structure as defined in Problem (9). This step in the algorithm aims to correct any potential misassignments of optimal organizational structures to firms, as we rely on information from their competitors. Given that this step is computationally intensive, we perform it only once. Following a single implementation, we continue with the previous method that utilizes information from competitors until convergence.[4]

After computing the optimal organizational structure, we update wages within the same iteration k. If the firm is initially multi-layer and we find a deviation that makes being

[4] Furthermore, we note that the algorithm does not improve in terms of correctly solving Problem (9) when we apply more times.

single-layer more profitable, we set managerial wages and employment to zero $w_{ijm}^{(k)} = 0$ and $n_{ijm}^{(k)} = 0$. For production workers, we set their wages and employment to the deviation values $w_{ijw}^{(k)} = w'$ and $n_{ijw}^{(k)} = n'$. If the firm is initially single-layer and we find a deviation that makes being multi-layer more profitable, we set both wages and employment to their deviation values $w_{ijo}^{(k)} = w'$ and $n_{ijo}^{(k)} = n'$ $\forall o \in \{w, m\}$. Note that the deviation may originate from competitors or the optimal choice, but it results in higher profitability in either case.

Overall, this numerical solution for the optimal organization structure of firms performs well. After convergence, we observe an optimal deviation from the equilibrium organizational choice for 0.34 percent of firms. This error arises because an optimal deviation implies that only one firm deviates from equilibrium while its local market competitors maintain constant labor demand. However, in practice, our algorithm solution often implies that more than one firm deviates, for instance, in markets with multiple firms. Therefore, implementing such an optimal deviation for those firms becomes impractical.

D.3 Algorithm with Minimum Wages

We solve the equilibrium with minimum wages using a shadow wage approach as in Berger et al. (2023b). This approach is useful to deal with non-market-clearing wages and assumes that workers at constrained firms perceive a lower wage whenever an excess of labor supply exists at the minimum wage. In particular, the shadow or perceived wage is the wage for which the firm's labor supply equals its labor demand at the minimum wage. This implies that the excess labor supply at the minimum wage is reallocated towards other firms.

We initialize the algorithm by guessing a vector of wages $\left\{w_{ijo}^{(0)}\right\}_{\forall ijo}$ such that the minimum wage is not binding for any of the firms in the first iteration. Thus, we set the initial vector of shadow wages $\left\{\tilde{w}_{ijo}^{(0)}\right\}_{\forall ijo}$ equal to the initial vector of wages. Consider iteration k:

1. **Compute labor supply**.
   Note that shadow wages are enough to get $\tilde{\mathbf{w}}_{jo}^{(k)}$, $\tilde{\mathbf{W}}_{o}^{(k)}$, and $\tilde{\mathbf{N}}_{o}^{(k)}$ from Equations (6) and (8). Then, we compute the labor supply to each firm $n_{ijo}^{s,(k)}$ by substituting the previous variables into Equation (7).

2. **Compute organizational choice.**

Using the shadow wage approach implies market clearing even with minimum wages. Thus, labor supply is equal to labor demand at the shadow wage $n_{ijo}^{d,(k)} = n_{ijo}^{s,(k)}$. Then, we use wages $\left\{w_{ijo}^{(k)}\right\}_{\forall ijo}$ and labor demands $\left\{n_{ijo}^{d,(k)}\right\}_{\forall ijo}$ to compute the optimal organizational choice $\ell^{(k)}$ for all firms using the same method that we describe in the previous subsection. Note that wages and employment change when we update the firm's optimal organizational structure, e.g., we set the wage and employment of managers to zero for single-layer firms: $w_{ijm}^{(k)} = 0$ and $n_{ijm}^{s,(k)} = n_{ijm}^{d,(k)} = 0$ if $\ell^{(k)} = 1$.

3. **Compute markdowns**.

   For each occupation, we use the optimal organization $\ell^{(k)}$ and labor demand $n_{ijo}^{d,(k)}$ of each firm to compute its marginal revenue product of labor $\text{mrpl}_{ijo}^{(k)} = \frac{\partial y(z,\ell^{(k)})}{\partial n_{ijo}}\Big|_{n_{ijo}^{d,(k)}}$. Then, we use this marginal product $\text{mrpl}_{ijo}^{(k)}$ and initial wages $w_{ijo}^{(k)}$ to compute:

   (a) *Minimum wage is not binding*: Whenever the firm's wage and marginal product are both above the minimum wage, we compute the payroll market share $s_{ijo}^{(k)}$ using wages $w_{ijo}^{(k)}$ and labor demand $n_{ijo}^{d,(k)}$ of all firms in market j. Then, we compute the firm's structural elasticity $\varepsilon_{ijo}^{(k)}$ and markdown $\mu_{ijo}^{(k)}$ from Equations (12)-(13).

   (b) *Minimum wage is binding, and the labor supply equals the labor demand at the minimum wage*: Whenever the minimum wage is binding and the firm's marginal product is above the minimum wage, we compute the firm's markdown from Equation (15).

   (c) *Minimum wage is binding, and the labor supply exceeds the labor demand at the minimum wage*: Whenever the minimum wage is binding and the firm's marginal product is lower than or equal to the minimum wage, we set $\mu_{ijo}^{(k)} = 1$.

4. **Compute wages.**

   For all firms, we update the occupation-specific wages as follows:

$$w_{ijo}'^{(k)} = \begin{cases} \max\left\{\underline{w}\ ,\ \mu_{ijo}^{(k)} \cdot \text{mrpl}_{ijo}^{(k)}\right\}, & \text{if} \quad n_{ijo}^{(k)} > 0, \\ 0, & \text{if} \quad n_{ijo}^{(k)} = 0. \end{cases}$$

   We use these updated wages to construct $\mathbf{w}_{jo}'^{(k)}$ and $\mathbf{W}_{o}'^{(k)}$ using Equation (8).

5. **Compute labor demand implied by minimum wages.**

   Here, we guarantee that the labor demand of constrained firms that face excess labor supply at the minimum wage is given by the inverse labor demand evaluated at the minimum wage. Particularly, we use the marginal product of labor $\text{mrpl}_{ijo}^{(k)}$ and updated wages $w_{ijo}^{\prime(k)}$ to compute:

   (a) *Minimum wage is not binding*: Whenever the firm's marginal product and wage are both above the minimum wage, the firm's labor demand coincides with the firm's labor supply $n_{ijo}^{\prime\, d,(k)} = n_{ijo}^{s,(k)}$.

   (b) *Minimum wage is binding, and the labor supply equals the labor demand at the minimum wage*: Whenever the minimum wage is binding and the firm's marginal product is above the minimum wage, the firm's labor demand coincides with the firm's labor supply $n_{ijo}^{\prime\, d,(k)} = n_{ijo}^{s,(k)}$.

   (c) *Minimum wage is binding, and the labor supply exceeds the labor demand at the minimum wage*: Whenever the minimum wage is binding and the firm's marginal product is lower than or equal to the minimum wage, we construct the labor demand of the firm from the employment level for which the minimum wage equals the marginal product of labor:

$$\underline{w} \;=\; \left.\frac{\partial y(z, \ell^{(k)})}{\partial n_{ijo}}\right|_{n_{ijo}^{\prime\, d,(k)}}.$$

6. **Update shadow wages.**

   The shadow wage is the wage that implies market clearing for all firms. That is, it does not coincide with the actual wage only for firms that face an excess of labor supply when they pay the minimum wage. We first use the updated employment levels $n_{ijo}^{\prime\, d,(k)}$ to update the market and aggregate employment levels from their definition:

$$\mathbf{n}_{jo}^{\prime\, d,(k)} := \left[\sum_{i=1}^{M_j} \left(n_{ijo}^{\prime\, d,(k)}\right)^{\frac{\eta_o+1}{\eta_o}}\right]^{\frac{\eta_o}{\eta_o+1}},$$

$$\mathbf{N}_o^{\prime\, d,(k)} := \left[\int_0^1 \left(\frac{\mathbf{n}_{jo}^{\prime\, d,(k)}}{B_{jo}}\right)^{\frac{\theta_o+1}{\theta_o}} dj\right]^{\frac{\theta_o}{\theta_o+1}}.$$

Then, we update shadow wages:

$$\tilde{w}_{ijo}^{(k+1)} = \left(\frac{1}{B_{jo}}\right)^{\frac{1+\theta_o}{\theta_o}} \left(\frac{n_{ijo}'^{\,d,(k)}}{\mathbf{n}_{jo}'^{\,d,(k)}}\right)^{\frac{1}{\eta_o}} \left(\frac{\mathbf{n}_{jo}'^{\,d,(k)}}{\mathbf{N}_o'^{\,d,(k)}}\right)^{\frac{1}{\theta_o}} \mathbf{W}_o'^{(k)}$$

7. **Iteration**.
Iterate over (1) to (6) until convergence of wages. Whenever $\max\left\{\text{abs}(w_{ijo}^{(k)} - w_{ijo}'^{(k)})\right\} >$ tol, we update wages with the following criterion:

$$\begin{aligned} \text{Unconstrained firms:} \quad & w_{ijo}^{(k+1)} = \rho w_{ijo}^{(k)} + (1-\rho) w_{ijo}'^{(k)} \quad \text{for} \quad \rho \in (0,1), \\ \text{Any constrained firm:} \quad & w_{ijo}^{(k+1)} = \underline{w}. \end{aligned}$$

E Appendix: Quantification of the Model

E.1 Targeted Moments

Table E.1 reports additional details to the quantification of the model parameters. In particular, it shows the model fit of each parameter to its most associated moment in the SMM estimation. The estimation brings about a close fit to the data, as it obtains an average absolute deviation of 5 percent between each model and data moment. The largest deviations occur in the span of control (7%) and wage gap parameters (8%).

E.2 Firm Substitutability Parameters

This section explains in detail the quantification of the parameters determining the structural labor supply elasticities: (η_o, θ_o).

Across-market elasticity

We use municipality-level data on wages and employment between 2002 and 2016 to estimate the across-market labor supply elasticity. The specification takes the following form:

$$\text{Log}\, w_{m,o,t} = \beta \,\text{Log}\, L_{m,o,t} + \alpha_{m,o} + e_{m,o,t}, \tag{C.26}$$

where $w_{m,o,t}$ is the average wage in municipality m, $L_{m,o,t}$ is the total employment in municipality m, and $\alpha_{m,o}$ are municipality fixed effects for each occupation. Because employment

Table E.1: Targeted Moments

Parameter Value	Description	Moment	Model	Data
A: Preferences				
ϕ_m	Labor disutility shifter: production workers	Average firm size	5.59	5.28
ϕ_w	Labor disutility shifter: managers	Ratio managers to workers	0.20	0.19
B: Firm Organization				
α	Span of control	Median span of control	3.41	3.14
φ_w	Worker efficiency	Mean wage of workers (€)	729	718
φ_m	Managerial efficiency	Wage gap managers and workers	0.79	0.73
σ_z	Std. Dev. firm TFP	Weighted mean HHI workers	0.18	0.19
C: Market Characteristics				
B_{ijw}	Amenities in small markets	Share workers in markets $M_j \leq 10$	0.12	0.12
Mass $m_j = 1$	Share single-firm markets	Mass single-firm markets	0.29	0.29
ζ_0	Scale Pareto distribution	Mean Nº firms	17.87	17.63
ζ_1	Shape Pareto distribution	Std. Dev. Nº firms	72.65	68.25
D: Firm Substitutability				
(θ_w, θ_m)	Across-market firm substitutability	Across-municipality LS elasticity	(1.52 , 0.92)	(1.46 , 0.92)
(η_w, η_m)	Within-market firm substitutability	Within-market LS elasticity	(19.97 , 6.10)	(19.97 , 6.10)

Note: The Table reports the vector of parameters estimated using the SMM approach and the calibrated firm distribution with their respective moment description and fit.

and wages are jointly determined in equilibrium, we use the following shift-share instrument for $L_{m,o,t}$:

$$\hat{L}_{m,o,t} = \sum_s \left(\underbrace{\frac{L_{i,m,s,o,2002}}{\sum_i L_{i,m,s,o,2002}}}_{\text{Industry-Municipality Share}} \times \underbrace{\sum_i L_{i,s,o,t}}_{\text{National Employment in Sector } s} \right). \tag{C.27}$$

The intuition for the instrument exploits across-municipality variation over time that stems from national employment shocks to sectors. The importance of each sectorial shift across municipalities depends on the sector's share in such municipality. Thus, municipalities vary in terms of exposure to the shift in sectors' employment. To explain why the instrument can be valid, we argue that multiple shifts to employment by sector, at the national level, are unrelated to local economic conditions. Hence, the national employment trends by sector exogenously adjust the local labor demand in this setup.

Regarding results, Table E.2 shows the estimates from the IV regression of Equation (C.26).

We find that the elasticity is lower for managers than for production workers. This suggests that it is more costly for managers to move across markets than for production workers.

Table E.2: Estimating the Across-Market Firm Substitutability Parameters

	(1) Production Workers	(2) Managers
Log employment	0.433***	1.008***
	(0.027)	(0.078)
Municipality FE	Yes	Yes
Observations	1,946	1,946
Implied Elasticity ($1/\beta$)	2.31	0.99
Inferred across-market substitutability (θ_o)	1.52	0.95

Note: The Table reports the estimates of the IV regression of Equation (17). Confidence intervals at the 95% level. The baseline period of the instrument is 2002, but we run the regression between 2009 and 2016 to exploit variation from the Great Recession. Standard errors are clustered at the municipality level. Source: 2002-2016, QP.

Within-market elasticity

To estimate the within-market labor supply elasticity, we use establishment-level information on wages and employment between 2002 and 2016. We compute total employment $L_{i,j,o,t}$ and average hourly real wages $w_{i,j,o,t}$ in occupation o for each establishment i in local labor market j at period t. We do not need to impute working hours because our database already provides this information. For each occupation o, we separately estimate the following regression:

$$logw_{i,j,o,t} = \beta \ logL_{i,j,o,t} + \mu_{j,o,t} + \upsilon_{i,j,o,t}, \tag{C.28}$$

We include local labor market-time fixed effects to isolate any time-varying shock in a given local labor market. Our goal is to estimate β, which represents the inverse of the within-market labor supply elasticity. The main threat to identification involves that the error $\upsilon_{i,j,o,t}$ captures establishment-time-specific shocks to labor demand and supply that are correlated with establishment size. In that case, the OLS estimate of β is biased. To address this problem, we additionally use a standard shift-share approach to simulate labor demand shocks that go back to **?** and are formalized more recently through the exogeneity of the shares (**?**) or the exogeneity of the shifts (**?**). The intuition of the instrument is that we exploit national trends in employment to predict establishment-level labor demand shocks. More concretely, we combine local shares and aggregate shifts to employment as follows:

$$\hat{L}_{i,j,o,t} = \underbrace{\frac{L_{i,j,o,2002}}{\sum_i L_{i,s,o,2002}}}_{\text{Firm's Employment Share in Sector } s} \times \underbrace{\sum_i L_{i,s,o,t}}_{\text{National Employment in Sector } s} . \tag{C.29}$$

We set 2002 as the initial year for the share component. Then, we measure the shares as the employment in occupation o in a given establishment located in j at t ($L_{i,j,2002,o}$) over the national level of employment for that occupation in sector s in 2002 ($L_{i,j,2002,o}$). Recall that our definition of local labor market j implicitly includes a sector, as it is the intersection between sector (s) and municipality (r) given an occupation o. We multiply this by the total employment of a given sector and occupation ($\sum_i^I L_{iso}$) every year after 2002 to predict current establishment employment according to national trends and initial shares. With the modified employment, we estimate Equation (C.28) by an instrumental variable with $\hat{L}_{i,j,o,t}$ as an instrument for $L_{i,j,o,t}$. In this estimation, we assume that the instrument is unrelated to unobserved constant or time-varying characteristics that affect specific establishments within the same industry, local labor market, and year.

Table E.3 shows the IV estimates of the reduced-form elasticities by occupations. We find that managers have a smaller labor supply elasticity than production workers, indicating that managers are less responsive to wage differentials across firms within the same market. Overall, our estimates are in the range of the literature. Most estimates based on inverse methods, i.e., estimating the inverse labor supply elasticity in the baseline specification, find estimates around 5.24 (**?**). Quantifying the model to county-level data in the U.S., Monte et al. (2018) finds a labor supply elasticity of 3.3. Using municipality-level German data, **?** find a labor supply elasticity of 5.5.

Table E.3: Estimating the Within-Market Firm Substitutability Parameters

	Production Workers		**Managers**	
Dependent Variable: Log Wage	**OLS**	**IV**	**OLS**	**IV**
Log employment	0.0500***	0.0623***	0.1639***	0.1645***
	(0.0002)	(0.0005)	(0.0005)	(0.0017)
Market-Year FE	Yes	Yes	Yes	Yes
Observations	2,580,623	459,960	1,036,216	119,165
Implied Elasticity ($1/\beta$)	19.96	16.05	6.10	6.08
Inferred within-market substitutability (η_o)	19.96	16.05	6.10	6.08

Note: The Table reports the estimates from regressing equation (19) by OLS and IV for each occupation. Standard errors in parentheses. Source: 2002-2016, QP.

E.3 Mass Layoff Shocks

This section explains in detail the definition and estimation of the effect of mass-layoff shocks on the municipality's employment and wages in the data. Then, we explain how we implement the same exercise in the model.

Definition

To identify mass layoffs in the Portuguese data, we consider sudden, sizable, and enduring reductions in the employment size of a prominent establishment within the regional economy. More precisely, we define that a region suffers a mass-layoff shock when it experiences a drop of 100 workers in any establishment for two consecutive years between 2004 and 2016.[5] All mass layoff events are aggregated at the municipal level to construct the treatment. So, we define the treated municipalities from the first time they experience a mass layoff in our sample period and those who never have one belong to the control group.

[5]We further restrict establishments with more than 1 percent of local employment in the baseline period, and we do not take into account plant closures to define mass layoffs.

Estimation

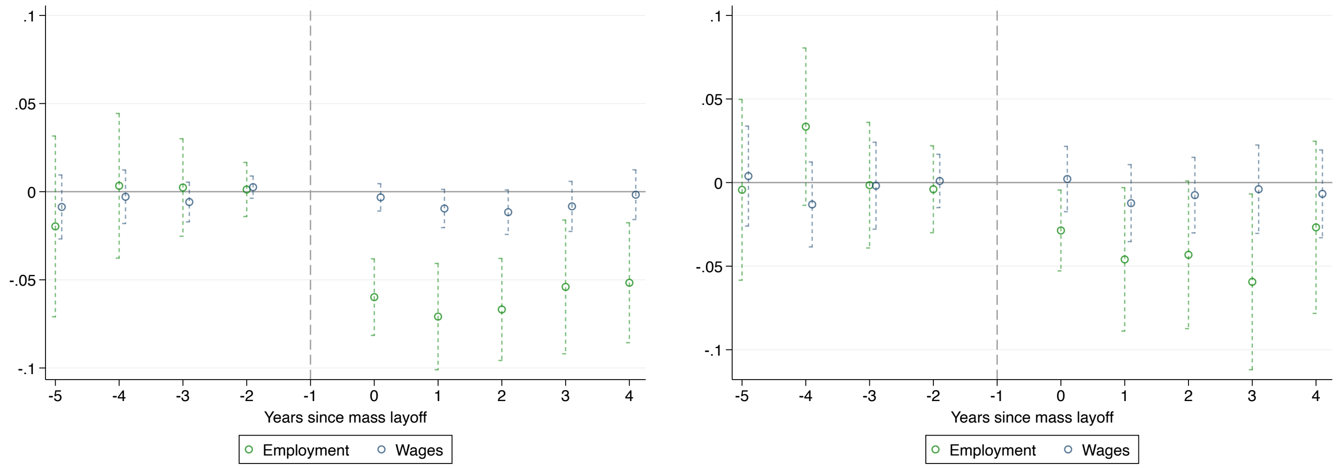
To quantify the impact on local employment and wages, we use an event-study specification that compares the changes in employment and wages of treated and control municipalities for managers and production workers following a mass layoff. Our specification takes the following form:

$$y_{mt} = \xi + \sum_{k=-5}^{5} \beta_k \mathbf{1}\{k = t - g\} + \gamma_m + \gamma_t + X_{it}'\theta + \epsilon_{mt}. \tag{C.30}$$

Here, the year that a mass layoff occurs in the establishment of a municipality is g, the years are t, and the event time indicators are k.[6] Year fixed effects (γ_t) and municipality fixed effects (γ_m) control for unobserved constant characteristics across all municipalities and within municipalities, respectively. The covariates X_{it} control for baseline characteristics regarding the size and structure of the cities interacted with time to flexibly control for time-varying variables. More precisely, these variables are the log of the municipality's employment, the share of manufacturing employment, the share of highly educated workers, the share of male workers, and the share of young workers. The parameters of interest are β_k, which come from k event time dummy variables. These dynamic treatment effects measure the effect on y relative to an omitted period, which is when $k = -1$. We use **?** estimator to control further for heterogeneous treatment effects across cohorts and aggregate all results across cohorts for the main coefficient shown in the main text. The main assumption needed in this setup is the conditional parallel trends assumption, stating that in the absence of mass layoffs, treated and control groups would evolve similarly once we net out baseline characteristics. Figure E.1 shows that pre-treatment coefficients are not statistically different from zero, suggesting that this assumption holds. In addition, Figure E.1a shows the dynamic post-treatment coefficient on employment and wages following a mass layoff shock. Overall, the most negative significant results happen on the employment margin, not on the wage margin, for both types of workers, similar to the findings of Gathmann et al. (2020) in Germany.

[6] We define the cohorts of treated municipalities from the first time, in our sample period, they had a mass layoff. Other mass layoffs may happen in the same municipality later on.

Figure E.1: Event Study Estimates by Occupation



(a) Production workers

(b) Managers

Note: These coefficients plot the estimated event study estimates using **?** estimator. We exploit 72 events of mass layoffs across time in our sample period. We identify mass layoffs as a drop of 100 workers in a given establishment for two consecutive years. Source: QP, 2004-2016.

Implementation in the model

We randomly group markets into municipalities and simulate two periods. The only difference between both periods is that the second contains a productivity shock randomly distributed across municipalities to firms that fulfill three characteristics that are informative of the firm's importance within the municipality and the size of the shock. First, we restrict the shock to multi-layer firms, as more than 95 percent of plants experiencing mass layoffs hire managers and production workers. Second, we exploit that the average firm carrying out a mass layoff had an average municipality employment share of 4.4 percent one year before the mass layoff. Thus, we condition the random shock on the sub-sample of firms whose municipality employment share falls within a specific range to match the same average size of shocked firms. Third, we choose the magnitude of the productivity shock to target that the average firm undergoing a mass layoff reduces its workforce by nearly 50 percent. Lastly, we run an OLS regression of employment and wages on a layoff dummy variable, controlling for municipality and time fixed effects.