# Monopsony Power and Firm Organization* 

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April 16, 2024
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#### Abstract

Labor market competition drastically differs for production workers and managers. For instance, in Portugal, there are half as many firms competing for managers as for production workers in the typical local labor market. Using administrative data from Portugal together with a general equilibrium model of oligopsony that incorporates minimum wages and management delegation, we show that monopsony power by firms leads to a welfare loss of $5.7 \%$ for production workers and $23.1 \%$ for managers relative to an efficient economy. Production workers bear smaller losses because they often work in markets with more competitor firms, view firms as closer substitutes, and are more affected by the minimum wage. The weak monopsony power of low-wage firms over production workers implies that raising the statutory minimum wage reduces overall welfare and affects managers through worker reallocation and delegation adjustments. Moving from the benchmark to an occupation-based minimum wage that optimally addresses monopsony power increases welfare by about $0.2 \%$ for both occupations.


Keywords: Monopsony power, firm organization, welfare, minimum wages.
JEL: D21, J21, J31, J42, O40

[^0]
## 1 Introduction

Limited labor market competition enables firms to offer lower wages, leading to lower employment, labor misallocation, and thereby welfare losses. ${ }^{1}$ This has influenced governments to implement policies that reduce firms' wage-setting power, such as minimum wages. However, uniform minimum wages fail to address the drastic differences in labor market competition across occupations, especially when occupational wage disparities are sizeable. For instance, in Portugal, half as many firms compete for managers as for production workers in the typical local labor market, as only some firms delegate decision-making to managers. ${ }^{2}$ This paper shows that the heterogeneity in monopsony power across these worker types matters when studying the welfare effects of monopsony power and minimum wage policies

To that end, we develop a general equilibrium model with firm-occupation-specific monopsony power and minimum wages. Using administrative data from Portugal, we show that monopsony power leads to a consumption equivalent welfare loss of 5.7 percent for production workers and 23.1 percent for managers relative to an efficient economy. The weak monopsony power of low-wage firms over production workers, who make up the majority of minimum wage earners, implies that raising the benchmark statutory minimum wage lowers overall welfare and affects managers through worker reallocation and management adjustments. Moving to an optimal occupation-based minimum wage increases welfare by about 0.2 percent for both occupations relative to the benchmark minimum wage.

To arrive at these conclusions, we first use matched employer-employee data from an annual census that covers the universe of private firms in Portugal between 2010 and 2016. We compute market concentration using the Herfindahl-Hirschman Index (HHI) for each occupation-based local labor market and document two novel stylized facts. First, we show that the average production worker and manager work in a market with an HHI equal to 0.19 and 0.27 , respectively. This corresponds to observing a market with five and three competitor firms, respectively. Indeed, we show that markets for managers are more concentrated

[^1]because, on average, a manager works in a local labor market where the number of active firms is only half that of the average production worker's market. This result suggests that firms have stronger monopsony power over managers. Second, we show that highly concentrated production workers' local labor markets have a higher share of firms that delegate decision-making to managers. In particular, an increase of 10 pp in the HHI is associated with an increase of 2.8 pp in the share of firms that delegate decisions to managers. This result indicates that the degree of competition in production workers' local labor markets affects the internal organization of firms and, thus, the allocation of managers across firms.

To map those facts into structural wage markdowns across occupations and perform counterfactual simulations, we develop a general equilibrium model of oligopsony that incorporates minimum wages and firm organization. The economy features a representative household for each occupation and a continuum of local labor markets, each with a finite number of firms. Households choose the labor supply to each individual firm for their respective occupations. Households view firms within the same and across distinct markets as imperfect substitutes due to preference heterogeneity, where the degree of substitutability is exogenously specific to each occupation. Firms exogenously differ in terms of productivity and the local labor market they inhabit. A firm's organizational decision involves adopting a single- or multilayer organizational structure. Single-layer firms decide how many production workers to hire. Multi-layer firms include an additional management layer and choose how many managers and production workers to hire. Managers enable firms to expand their workforce, yet it comes at higher overhead costs, and thus, only the most productive firms adopt a multi-layer structure. Regarding monopsony power, imperfect firm substitutability and firm granularity imply that firms face upward-sloping labor supply curves and internalize their size within the local labor market, which leads to firm-occupation-specific markdowns.

We estimate the model using the Simulated Method of Moments (SMM) to fit moments of firm organization, wages, and market concentration in Portugal. For each occupation, the firm substitutability parameters determine the within-market and across-market labor supply elasticities. We exploit the correlation between employment and wages at the establishment level, controlling for market unobserved heterogeneity, to calibrate the within-market elasticities. Moreover, we adopt an indirect inference approach to estimate the across-market elasticities from plausibly exogenous labor demand changes at the municipality level, which
we generate with a Bartik-type instrument that exploits the exposure of each municipality to national sector employment trends during the Great Recession. The model reproduces the empirical distribution of wages across workers and the distribution of employment across markets. Moreover, it fits average levels of market concentration, firm organization, and the positive relationship between market concentration and the share of production workers at multi-layer firms. To validate the estimated elasticities, we conduct an event study analysis that quantifies the effect of mass-layoff shocks on employment and wages at the municipality level. For each occupation, our model captures that mass layoffs slightly decrease the municipality's average wage and significantly decrease the municipality's employment.

To quantify the effect of monopsony power on aggregate outcomes, we compare the benchmark equilibrium with a counterfactual efficient economy where we exogenously set wages to the marginal product of labor. In the benchmark economy, we estimate that the average manager and production worker bear a wage markdown of 23.1 and 10.6 percent, respectively. Firms generally exert wider wage markdowns on managers because managers (i) often work in markets with fewer competitor firms, (ii) the minimum wage is more likely to be binding for production workers, and (iii) have lower across and within market elasticities. Regarding the lower elasticities of managers, our result aligns with evidence on the lower labor supply elasticity of top earners (Langella and Manning, 2021) and workers performing non-routine cognitive tasks relative to workers performing routine or non-routine manual tasks (Bachmann et al., 2022). Moreover, we show that production workers are more likely to switch firms than managers, as they are younger and more likely to have temporary contracts.

We derive three main results when comparing the benchmark and efficient economy. First, the efficient economy increases the managerial wage premium by 11.8 percent. Second, the share of multi-layer firms decreases by 11.2 percent in the efficient economy because rising wages make managerial delegation more costly, especially for medium-productivity firms. Among these, the share of multi-layer firms decreases by 12.6 to 44.4 percent. The result is an increase in manager concentration at the most productive firms and these firm expanding also their hiring of production workers and their production. Third, the efficient economy provides a consumption-equivalent welfare gain of 5.7 percent for production workers and 23.1 percent for managers, as they benefit from the efficiency gains and the redistribution of profits to labor income. We show that accounting for the endogenous drop in managerial
delegation in an efficient economy avoids overestimating the welfare gains of production workers by 0.6 pp and managers by 2.1 pp . The reason is that the decrease in managerial delegation, which stems from removing monopsony power, reduces the consumption level of managers due to lower labor demand. Moreover, the induced reallocation of production workers to other firms increases their disutility from labor supply.

To assess the implications for the design of optimal minimum wage policies to alleviate monopsony power, we first analyze the Portuguese reforms that increased the real minimum wage by ten percent between 2016 and 2019. The weak monopsony power of low-wage firms over production workers, who make up the majority of minimum wage earners, undermines the effectiveness of this policy. We show that the minimum wage increase deteriorates production workers' welfare by 0.7 percent, mainly due to their disemployment effects. Regarding managers, the increase in the minimum wage maintains their welfare with two opposing effects. On the one hand, it decreases the demand for managers in medium-productivity firms that reduce their workforce of production workers. On the other hand, it increases the demand for managers in high-productivity firms due to the reallocation of production workers from other firms.

Second, we quantify the welfare effects of designing optimal occupation-based minimum wages to address the heterogeneous markdowns across worker types. ${ }^{3}$ First, we find that the welfare gain of managers is hump-shaped in increasing the minimum wage just for managers while keeping the baseline minimum wage for production workers (525€). At most, this policy yields a welfare gain for managers of about 0.5 percent when their minimum wage is 56 percent of their mean wage $(900 €)$. However, it decreases the welfare of production workers by 0.1 percent. Second, we compute the combination of occupation-based minimum wages that generate a Pareto optimal improvement relative to the benchmark economy. Setting the minimum wage for production workers at 63 percent of their mean wage $(460 €)$ and the one for managers at 50 percent of their mean wage $(790 €)$ increases their welfare level by 0.3 and 0.2 percent, respectively. That is, even an optimal occupation-based minimum wage recovers less than 5 percent of the welfare loss from monopsony power for each occupation.

[^2]Literature. This paper contributes to the literature on oligopsonistic labor markets and how this affects the overall economy (Bhaskar et al., 2002; MacKenzie, 2021; Berger et al., 2022; Deb et al., 2022; Berger et al., 2023a; Jarosch et al., 2023; Azkarate-Askasua and Zerecero, 2023). Using models where labor market power arises from firm granularity and imperfect firm substitutability, they study the effect of labor market power on wages, efficiency, and welfare. Our main theoretical contribution to this literature is to study the effect of monopsony power on these outcomes through the organization of work within firms. The distinctive mechanism in our model is that firms make organizational decisions that endogenously contribute to markdown heterogeneity across worker types. Our main quantitative contribution is to show that the large heterogeneity in wage markdowns between managers and production workers matters to explain the managerial wage premium and the welfare losses from monopsony power.

We connect to the literature that studies the effect of minimum wage policies in models with imperfect labor market competition (Bamford, 2021; Ahlfeldt et al., 2022; Hurst et al., 2022; Karabarbounis et al., 2022; Drechsel-Grau, 2023). We build our framework on Berger et al. (2023b), which studies the effect of minimum wages on efficiency and welfare in an oligopsonistic environment with firm and worker heterogeneity. Our main contribution is to allow for occupation-specific markdowns and imperfect substitutability across worker types in production. This matters for adding three findings to this literature. First, we rationalize that minimum wages affect the employment and wage distribution of managers by affecting production workers. Second, we show that raising the minimum wage decreases the share of firms that delegate to managers, which is crucial to explain the reallocation of managers towards high-productivity firms. Third, we show that adopting an occupation-based minimum wage can provide welfare gains.

This paper also contributes to the literature on production organization (Garicano and RossiHansberg, 2006; Caliendo and Rossi-Hansberg, 2012). Several studies build on this model to analyze firm-size distortions (Garicano et al., 2016; Tamkoç, 2022), the adoption of information and technological capital (Mariscal, 2020), the misallocation of labor in developing countries (Grobovsek, 2020), and technological adoptions across urban areas (Santamaria, 2023). Contemporaneously to our work, Lawson et al. (2023) studies the impact of minimum wages on productivity through firm organization in a perfectly competitive framework. To
the best of our knowledge, we are the first to incorporate monopsony power in a general equilibrium model with managerial delegation choices. This adds two contributions to this literature. First, delegation choices help to explain the degree of monopsony power over managers and production workers. Second, we show that lower competition in production workers' markets incentivizes firms to delegate tasks to managers.

We also contribute to the literature that studies the misallocation of labor across firms (Hsieh and Klenow, 2009; Bartelsman et al., 2013; Davis et al., 2014; Garcia-Santana and Pijoan-Mas, 2014; Heise and Porzio, 2023). We show that large firms mainly restrict managerial employment relative to the efficient level in Portugal, leading to an inefficiently high managerial span of control and to an inefficiently high share of multi-layer firms.

Finally, we relate to the empirical literature on monopsony and labor market concentration (Martins, 2018; Azar et al., 2020; Benmelech et al., 2020; Rinz, 2022; Azar et al., 2022; Bassanini et al., 2023; Dodini et al., 2023). We bring two new stylized facts to this literature. First, we document that managerial markets display higher levels of market payroll concentration relative to the markets of production workers. Second, we highlight that this heterogeneity stems from managerial markets having fewer firms and because managers sort into more concentrated markets.

## 2 Stylized Facts

This section documents two stylized facts that motivate the relationship between firms' organizational decisions and labor market concentration. First, the average production worker and manager work in a market with an HHI equal to 0.19 and 0.27 , respectively. Second, the share of production workers that work in multi-layer establishments, i.e., establishments that add a management layer to their organization, is higher when their local labor market is more concentrated.

### 2.1 Data

Our primary data source is Quadros de Pessoal (QP), an annual census of private sector employees conducted by the Portuguese Ministry of Employment. This census provides
matched employer-employee data with information on employment, monthly wages, occupation, industry, and municipality for all private firms based in Portugal with at least one worker. Our sample period covers from 2010 to 2016. We explain here the main aspects of the sample and relegate the details to Appendix B.

We define two broad occupations: managers and production workers. We classify labor markets for each occupation based on their geography (municipality) and industry (2-digit NACE). This classification stems from the fact that workers are more attached to their current labor market because of the imperfect substitutability of skills across jobs and sectors, as well as imperfect geographical mobility (Neal, 1995; Kambourov and Manovskii, 2009; Sullivan, 2010; Kennan and Walker, 2011; Monte et al., 2018).

We assign workers to each occupation following a hierarchical classification similar to Caliendo et al. (2020). By Portuguese law, firms must assign workers to hierarchic categories that allow us to distinguish between two layers within each firm (see Table A.1). We exclude CEOs and assign middle managers, supervisors, team leaders, and top managers to the management layer. The distinctive feature of managers is that they guide groups of employees in their tasks. In our sample, nearly one-fifth of employees are managers, who are mostly supervisors, team leaders, or middle managers (see Table A.2). We group the remaining categories as production workers, which range from non-skilled to higher-skilled professionals. ${ }^{4}$

### 2.2 Payroll Concentration and Firm Organization

The level of market payroll concentration in a local labor market, which represents how much of the total market payroll belongs to a few establishments, is a standard proxy for the degree of monopsony power that establishments hold in such a market. ${ }^{5}$ We measure payroll concentration in each local labor market using the HHI. This index equals a weighted average payroll share of establishments within the market. Thus, an increase in the HHI

[^3]Table 1: Market Concentration by Occupation

|  | Managers | Production Workers |
| :--- | :---: | :---: |
|  | Mean | Mean |
| ${\text { Max } s_{i j}}^{\mathrm{HHI}_{j}}$ | 0.38 | 0.30 |

Source: Elaboration based on Quadros de Pessoal.
Note: The first row reports the employment-weighted mean of the maximum payroll share across local labor markets. The second row of the Table reports the employment-weighted mean of the HHI across local labor markets.
reflects higher market concentration because fewer establishments accumulate a greater share of the market payroll.

Fact \#1: Managers work in more concentrated labor markets. In Table 1, we document that the average manager works in a market whose HHI is eight percentage points higher than the average production worker. ${ }^{6}$ Specifically, the employment-weighted average HHI is 0.27 for managers and 0.19 for production workers. To provide context for these numbers, one would observe a similar concentration level with three and five equally-sized establishments, respectively. Table 1 also reports the employment-weighted average of the maximum payroll share across local labor markets. The average largest establishment in a local labor market accumulates about eight percentage points more market payroll in the market of managers than in the market of production workers.

Two underlying forces determine the heterogeneity in market concentration between both occupations. First, the heterogeneity in the distribution of payroll concentration across markets. Second, the heterogeneity in employment sorting across these market types. We display both channels in Figure 1, which plots the estimated kernel density of the market level HHI for each occupation and their respective cumulative share of employment.

First, we find that managers work in more concentrated markets because the marketconcentration distribution for managers is shifted to the right relative to production workers (see Figure 1a). We use a standard decomposition that breaks the HHI into two elements

[^4]Figure 1: Distribution of HHI by Occupation


Source: Elaboration based on QP.
Note: The Graph plots the kernel density of the payroll HHI across local labor markets for managers (dashed line) and production workers (solid line). Moreover, the graph plots the cumulative share of employment with respect to the payroll HHI.
to understand what makes managerial markets more likely to be concentrated (see Appendix B. 4 for details). The first element involves the number of establishments in each market. All else being constant, increasing the number of establishments lowers the average establishment size in the market. The second element entails the dispersion level of payroll shares across establishments relative to the case in which they hold identical shares. All else being constant, increasing the dispersion in payroll shares leads to greater payroll concentration. Table 2 reports the results from decomposing the unweighted average of the HHI for both occupations. ${ }^{7}$ We find that the entire gap of nine percentage points stems from the fact that managerial markets have fewer establishments. In particular, the markets of production workers tend to have almost two times as many establishments as the markets of managers (see Table A.6). This fact emphasizes the importance of modeling establishments' decision-making regarding managerial delegation, as only some establishments choose to hire managers, leading to more concentration in their labor markets.

[^5]Table 2: Decomposition of the Average HHI across Occupations

|  | Component: No establishments | Component: Dispersion in shares | Mean HHI |
| :--- | :---: | :---: | :---: |
| (1) Managers | 0.57 | 0.08 | 0.65 |
| (2) Production Workers | 0.47 | 0.09 | 0.56 |
| (1)-(2) Gap | 0.10 | -0.01 | 0.09 |

Note: The Table reports the contribution of each channel to the unweighted average HHI level across local labor markets for each occupation. The first column reports the contribution of the number of establishments $(1 / M)$. In contrast, the second column shows the contribution of the dispersion in payroll shares relative to the symmetric case $\left(\sum_{i=1}^{M}\left(s_{i}-\frac{1}{M}\right)^{2}\right)$.

Figure 2: Market Concentration and Multi-layer Firms


Source: Elaboration based on QP.
Note: The Figure plots the employment-weighted average share of multi-layer firms across local labor markets that differ in the level of HHI. In particular, we compute the share of multi-layer firms and the HHI for each local labor market of production workers. We split the distribution of the HHI into 20 cells of length 0.05 . In each cell, we take the employment-weighted mean of the share of multi-layer firms across markets.

Second, we find that managers are more likely to sort into highly concentrated markets (see Figure 1b). For instance, nearly 70 percent of production workers are in markets with an HHI below 0.20 , whereas only 60 percent of managers work in such markets. Most employees in both occupations work in a handful of markets with relatively low concentration levels. However, the proportion of managers in those markets is comparatively smaller. Different sorting patterns emphasize the need to model labor allocation across markets.

Fact \#2: More firms delegate decision-making to managers in highly concentrated production workers' markets. Figure 2 displays the weighted average share of multi-layer establishments across the markets of production workers. We rank each market by their HHI and use the number of employees as weights. When concentration rises in production workers' markets, the share of multi-layer firms increases. ${ }^{8}$ An increase of 10 pp in the HHI is associated with an increase of 2.8 pp in the share of multi-layer firms. This suggests a tight relationship between the internal organization of firms and the level of payroll concentration in the labor markets where they operate. Two complementary stories with different economic implications may explain this result. On the one hand, this relationship may reflect the tendency of high-productivity firms to adopt a multi-layer structure as they expand, consequently contributing to increased market concentration. On the other hand, it may reflect that small markets with low competition allow firms, even when their productivity is low, to easily attract workers and expand their organization because mobility frictions prevent production workers from relocating to other firms.

## 3 Model

This section presents a general equilibrium model that incorporates firm organization, oligopsonistic labor markets, and minimum wages. The model considers two occupations, managers and production workers, each with heterogeneous labor disutility costs and firm substitutability parameters. For each occupation, there is a household that makes consumption choices and decides the labor supply to each firm. Firms are heterogeneous in productivity and the local labor market they inhabit. Regarding their organization, firms have a layer of production workers and choose whether to add a management layer. Then, they choose the number of workers in each layer. Firms have monopsony power and face a minimum wage when making employment choices, where wage markdowns are firm and occupation-specific.

[^6]
### 3.1 Environment

The internal organization of firms is related to labor market concentration and, potentially, to monopsony power. We stand out two reasons to analyze this relationship through the lens of a general equilibrium model. First, the fact that managerial markets are more concentrated suggests that firms have greater monopsony power over managerial wages. Yet, these market categorizations do not capture the entire set of potential employers, as workers flow across industries and regions. Thus, to quantify monopsony power, it is necessary to measure the extent to which workers find it costly to switch firms both within the same and across different markets. Second, the fact that multi-layer firms contribute more to production workers' employment when their markets are more concentrated suggests that monopsony power affects managerial delegation choices. Modeling the relationship between both variables and performing counterfactual simulations solve the identification problem arising from the confounding that delegation may also contribute to market concentration, as multi-layer firms are usually bigger. Next, we describe the environment of the model in detail.

Agents. The economy is populated by two households, indexed by their permanent occupation type $o \in\{w, m\}$, and a continuum of firms. The two households differ in how substitutable they view different firms within the same and across different markets, as well as in terms of disutility costs for each unit of work. Firms are exogenously heterogeneous in two dimensions. First, they belong to a continuum of local labor markets $j \in[0,1]$, i.e., intersections between region and industry, where each local labor market contains a finite number of firms indexed by $i \in\left\{1, \ldots, M_{j}\right\}$. Second, firms differ in productivity $z_{i j}$, drawn from a standard log-normal distribution with standard deviation $\sigma_{z}$.

Goods and Technology. Firms use labor to produce a tradable good in a perfectly competitive national market whose price we normalize to one. We assume that there are two types of labor: production workers and managers. Production workers are essential for production, while managers are optional. We assume each firm chooses between two types of organizations, which vary in the number of layers, $\ell \in\{1,2\}$.

The first type of firm organization is the single-layer firm $(\ell=1)$. Single-layer firms use $n_{w}$
units of labor of production workers to produce according to the following technology:

$$
\begin{equation*}
y(z, 1)=z \varphi_{w} n_{w}^{1-\alpha} . \tag{1}
\end{equation*}
$$

The parameter $\varphi_{w}$ stands for the efficiency of labor of production workers. This technological specification includes the essential features of the production technology in the standard model of Lucas (1978). Namely, firms are heterogeneous in productivity and face diminishing returns to scale. The parameter $1-\alpha \in(0,1)$ determines the strength of diminishing returns and, thus, limits the number of production workers of single-layer firms.

Alternatively, firms may choose to be multi-layer organizations $(\ell=2)$ to manage a larger workforce. In this case, firms additionally include a managerial layer and use the labor of both managers $n_{m}$ and production workers $n_{w}$ to produce output. The technology of multi-layer firms is given by:

$$
\begin{equation*}
y(z, 2)=z \varphi_{m} n_{m}^{(1-\alpha) \alpha} n_{w}^{1-\alpha} . \tag{2}
\end{equation*}
$$

The parameter $\varphi_{m}$ reflects how much managers enhance the productivity of production workers relative to the single-layer organization. Under this technological specification, managerial delegation allows firms to increase their workforce by dampening the diminishing returns to the labor of production workers. In addition, the parameter $\alpha$ embeds in a simple manner all the technical reasons that influence the span of control of managers, as it is the only technological parameter that is informative of the ratio of the marginal labor productivity between the two occupations.

Overall, this technological specification abstracts from the micro-foundations in the theory of firm organization, such as those of the knowledge-based hierarchy literature (Garicano, 2000; Garicano and Rossi-Hansberg, 2006). Nevertheless, it facilitates the quantification of the model and considers the main organizational trade-off. That is, adding a managerial layer permits the firm to manage a larger workforce, but it comes at the expense of higher costs.

Households. Each household type $o \in\{w, m\}$ chooses the measure of workers to supply to
each firm $n_{i j o}$ and consumption of each good $c_{i j o}$ to maximize their utility:

$$
\begin{equation*}
\mathcal{U}_{o}=\max _{n_{i j o}, c_{i j o}} \quad \mathbf{C}_{o}-\phi_{o} \frac{\mathbf{N}_{o}^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}}, \tag{3}
\end{equation*}
$$

subject to the household's budget constraint:

$$
\begin{equation*}
\mathbf{C}_{o}=\int_{0}^{1} \sum_{i=1}^{M_{j}} w_{i j o} n_{i j o} d j \tag{4}
\end{equation*}
$$

where we define the aggregate consumption and labor supply indexes as

$$
\begin{aligned}
& \mathbf{C}_{o}:=\int_{0}^{1} \sum_{i=1}^{M_{j}} c_{i j o} d j \\
& \mathbf{N}_{\mathbf{o}}:=\left[\int_{0}^{1}\left(\frac{\mathbf{n}_{\mathbf{j o}}}{B_{j o}}\right)^{\frac{\theta_{o}+1}{\theta_{o}}} d j\right]^{\frac{\theta_{o}}{\theta_{o}+1}} \mathbf{n}_{\mathbf{j o}}:=\left[\sum_{i=1}^{M_{j}} n_{i j o}^{\frac{\eta_{o}+1}{\eta_{o}}}\right]^{\frac{\eta_{o}}{\eta_{o}+1}}, \quad \eta_{o}>\theta_{o}>0 .
\end{aligned}
$$

The parameter $\gamma$ stands for the aggregate Frisch elasticity of households, $\phi_{o}$ is a labor disutility shifter that is specific to each occupation, and $B_{j o}$ is a market amenity shifter. We follow closely the consumption and labor supply structure of Berger et al. (2022). That is, we assume that consumption goods are perfectly substitutable, but households view firms as imperfect substitutes in terms of non-wage characteristics. In particular, each firm faces an occupation-based upward-sloping labor supply curve with two elasticities of substitution $\theta_{o}>0$ and $\eta_{o}>0$. The parameter $\theta_{o}$ regulates the degree of substitutability of firms in distinct markets and, thus, captures the costs of moving across markets or idiosyncratic tastes for the market. If these costs decrease $\left(\theta_{o} \uparrow\right)$, workers find it easier to substitute firms across markets and become more responsive to market wage differentials. The parameter $\eta_{o}$ regulates the degree of substitutability of firms within the same market, thus capturing features such as commuting costs, search costs, or idiosyncratic tastes for the firm. As these costs decrease $\left(\eta_{o} \uparrow\right)$, workers find within-market, across-firm substitutability easier and become more responsive to wage differentials across firms in the same market. We refer to $\eta_{o}$ and $\theta_{o}$ as the within- and across-market firm substitutability parameters. Critical to the main conclusions of the theory, we impose the following assumption:

Assumption 1. Across-market mobility costs are higher than within-market mobility costs:

$$
\begin{equation*}
\eta_{0} \geq \theta_{0}, \forall o \in\{0,1\} \tag{5}
\end{equation*}
$$

In other words, we assume that both household types find firms within the same market as closer substitutes than firms in different markets. Under this assumption, larger firms hinder the reallocation of their workers to other firms because their workers have fewer alternatives within the same market and, thus, need to move to other markets to find more potential employers. As a result, employees become less responsive to the wage policy of the firm, which in turn provides greater monopsony power to larger firms.

Linear utility in consumption implies that the aggregate labor supply in occupation $o$ is:

$$
\begin{equation*}
\mathbf{N}_{o}=\left(\frac{\mathbf{W}_{o}}{\phi_{o}}\right)^{\gamma} \tag{6}
\end{equation*}
$$

and the labor supply curve of occupation $o$ to firm $i$ in market $j$ is:

$$
\begin{equation*}
n_{i j o}=B_{j o}^{1+\theta_{o}}\left(\frac{w_{i j o}}{\mathbf{w}_{j o}}\right)^{\eta_{o}}\left(\frac{\mathbf{w}_{j o}}{\mathbf{W}_{o}}\right)^{\theta_{o}} \mathbf{N}_{o} \longleftrightarrow \underbrace{w_{i j o}=\left(\frac{1}{B_{j o}}\right)^{\frac{1+\theta_{o}}{\theta_{o}}}\left(\frac{n_{i j o}}{\mathbf{n}_{j o}}\right)^{\frac{1}{\eta_{o}}}\left(\frac{\mathbf{n}_{j o}}{\mathbf{N}_{o}}\right)^{\frac{1}{\theta_{o}}} \mathbf{W}_{\mathbf{o}}}_{\text {Inverse labor supply curve } \forall i j o}, \tag{7}
\end{equation*}
$$

where we define the market wage index $\mathbf{w}_{j o}$ and the aggregate wage index $\mathbf{W}_{o}$ as

$$
\begin{equation*}
\mathbf{w}_{j o}:=\left[\sum_{i \in j} w_{i j o}^{1+\eta_{o}}\right]^{\frac{1}{1+\eta_{o}}} \quad \mathbf{W}_{\mathbf{o}}:=\left[\int_{0}^{1}\left(B_{j o} \mathbf{w}_{\mathbf{j o}}\right)^{1+\theta_{o}} d j\right]^{\frac{1}{1+\theta_{o}}} . \tag{8}
\end{equation*}
$$

We derive the labor supply system of equations given by (6) and (7) in Appendix C.1.
Firms. Firms choose the organizational structure to maximize profits, which consists of choosing whether to add a management layer:

$$
\begin{equation*}
\pi(z)=\max _{\ell}\{\pi(z, \ell)\}_{\ell=1}^{2} \tag{9}
\end{equation*}
$$

In addition, firms make employment choices given upward-sloping labor supply curves and oligopsonistic competition for labor. We assume firms are constrained by a minimum wage. Infinitesimal with respect to the economy, firms take the aggregate disutility of labor supply $\mathbf{N}_{o}$ and aggregate wages $\mathbf{W}_{o}$ as given. However, non-atomistic to the local market, each firm internalizes the impact of the employment decisions of all firms in the market, including itself, on the labor supply curve it faces.

When firms adopt a single-layer structure, they also choose the measure of production workers $n_{i j w}$ to maximize profits, given the employment policies of their local competitors, $n_{-i j w}^{*}$. In particular, they solve:

$$
\begin{equation*}
\pi(z, 1)=\max _{n_{i j w}} y(z, 1)-w_{i j w}\left(n_{i j w}, n_{-i j w}^{*}, \mathbf{N}_{w}, \mathbf{W}_{w}\right) n_{i j w} \tag{10}
\end{equation*}
$$

subject to the inverse labor supply curve of production workers and minimum wages:

$$
\left.\begin{array}{rl}
w_{i j w}\left(n_{i j w}, n_{-i j w}^{*}, \mathbf{N}_{w}, \mathbf{W}_{w}\right) & =\left(\frac{1}{B_{j w}}\right)^{\frac{1+\theta_{o}}{\theta_{o}}}\left(\frac{n_{i j w}}{\mathbf{n}_{j w}\left(n_{i j w}, n_{-i j w}^{*}\right)}\right)^{\frac{1}{\eta_{w}}}\left(\frac{\mathbf{n}_{j w}\left(n_{i j w}, n_{-i j w}^{*}\right)}{\mathbf{N}_{w}}\right)^{\frac{1}{\theta_{w}}} \mathbf{W}_{w} \\
\mathbf{n}_{j w}\left(n_{i j w}, n_{-i j w}^{*}\right) & =\left[n_{i j w}^{\frac{1+\eta_{w}}{\eta_{w}}}+\sum_{k \neq i} n_{k j w}^{*} \frac{1+\eta_{w}}{\eta_{w}}\right.
\end{array}\right]^{\frac{\eta_{w}}{1+\eta_{w}}}, ~\left(w_{i j w} \geq \underline{w}\right.
$$

When firms adopt a multi-layer structure, they also choose the measure of production workers $n_{i j w}$ and managers $n_{i j m}$ to maximize profits, given the employment policies of their local competitors, $\left(n_{-i j w}^{*}, n_{-i j m}^{*}\right)$. Their maximization problem is:

$$
\begin{equation*}
\pi(z, 2)=\max _{n_{i j w}, n_{i j m}} y(z, 2)-\sum_{o \in\{w, m\}} w_{i j o}\left(n_{i j o}, n_{-i j o}^{*}, \mathbf{N}_{o}, \mathbf{W}_{o}\right) n_{i j o} \tag{11}
\end{equation*}
$$

subject to the inverse labor supply curve of both occupations:

$$
\begin{aligned}
& w_{i j o}\left(n_{i j o}, n_{-i j o}^{*}, \mathbf{N}_{o}, \mathbf{W}_{o}\right)=\left(\frac{1}{B_{j o}}\right)^{\frac{1+\theta_{o}}{\theta_{o}}}\left(\frac{n_{i j o}}{\mathbf{n}_{j o}\left(n_{i j o}, n_{-i j o}^{*}\right)}\right)^{\frac{1}{\eta_{o}}}\left(\frac{\mathbf{n}_{j o}\left(n_{i j o}, n_{-i j o}^{*}\right)}{\mathbf{N}_{o}}\right)^{\frac{1}{\theta_{o}}} \mathbf{W}_{o} \\
& \mathbf{n}_{j o}\left(n_{i j o}, n_{-i j o}^{*}\right)=\left[n_{i j_{o}}^{\frac{1+\eta_{o}}{\eta_{o}}}+\sum_{k \neq i} n_{k j o}^{*} \frac{1+\eta_{o}}{\eta_{o}}\right]^{\frac{\eta_{o}}{1 \eta_{o}}} \\
& w_{i j w} \geq \underline{w}, \quad \forall o \in\{w, m\} .
\end{aligned}
$$

Given the occupation $o \in\{w, m\}$ and the number of layers $\ell \in\{1,2\}$, the solution to the firm's problem has three cases. First, the minimum wage is not binding. Second, the minimum wage is binding, and labor demand equals the labor supply curve. Third, the
minimum wage is binding and labor supply exceeds labor demand. We summarize the system of first-order conditions for each case as follows.

Case I: The minimum wage is not binding. Firms choose the level of employment in occupation $o \in\{w, m\}$ for which the marginal cost of labor equals its marginal product (see Figure A.1).

$$
\begin{equation*}
w_{i j o}^{*}=\left.\mu_{i j o} \frac{\partial y(z, \ell)}{\partial n_{i j o}}\right|_{n_{i j o}^{*}}, \quad \mu_{i j o}=\frac{\varepsilon_{i j o}}{\varepsilon_{i j o}+1} \in(0,1), \quad \varepsilon_{i j o}=\left[\frac{\partial \log w_{i j o}}{\partial \log n_{i j o}}\right]^{-1} . \tag{12}
\end{equation*}
$$

Equation (12) determines that the marginal productivity of labor is higher than the wage payment for each occupation (see Appendix C. 2 for complete derivations). As in the classical monopsony environment (Manning, 2013), the marginal cost of labor is equal to both the wage and the additional cost of increasing wages because firms internalize upward-sloping labor supply curves. Hence, there is a wedge between wages and the marginal product of labor $\mu_{i j o}<1$. In addition, firm granularity within each local labor market implies that wage markdowns and the labor supply elasticity are firm-specific because firms internalize the impact of their relative market size on the labor supply curve they face. In particular, the structural labor supply elasticity has a closed-form solution and is given by:

$$
\begin{equation*}
\varepsilon_{i j o}\left(s_{i j o}\right)=\left[\frac{1}{\eta_{o}}+\left(\frac{1}{\theta_{o}}-\frac{1}{\eta_{o}}\right) \frac{\partial \log \mathbf{n}_{j o}}{\partial \log n_{i j o}}\right]^{-1}=\left[\frac{1}{\eta_{o}}+\left(\frac{1}{\theta_{o}}-\frac{1}{\eta_{o}}\right) s_{i j o}\right]^{-1}, \tag{13}
\end{equation*}
$$

where $s_{i j o}$ stands for the payroll share of firm $i$ in market $j$ :

$$
\begin{equation*}
s_{i j o}:=\frac{w_{i j o} n_{i j o}}{\sum_{i \in j} w_{i j o} n_{i j o}} \tag{14}
\end{equation*}
$$

Note that firms face an elasticity that is a function of their payroll share for each occupation. Since we assume $\eta_{0} \geq \theta_{0}$, larger firms face lower labor supply elasticities and exert wider markdowns. On the one hand, consider the monopsony case with only one firm in the market, i.e., $s_{i j o}=1$. If a monopsonist makes an additional hire, it understands that it has to attract workers from other markets. Hence, it faces the across-market labor supply elasticity $\theta_{i j o}$. On the other hand, consider an atomistic firm $\left(s_{i j o} \simeq 0\right)$. This firm internalizes that a marginal hire attracts workers from the same market, as its decision has a negligible effect on market employment. Therefore, it faces the within-market labor supply elasticity $\eta_{i j o}$.

The model explicitly tells apart the forces shaping wage dispersion across occupations. The first source of dispersion comes from differences in marginal productivity, which depends on organizational choices. The second source of dispersion comes from differences in markdowns. Firms have occupational-based monopsony power because they may have different sizes in each local labor market $\left(s_{i j o}\right)$ and because firm substitutability may differ across occupations $\left(\eta_{o}, \theta_{o}\right)$.

Case II: The minimum wage is binding, and labor supply equals labor demand. The minimum wage is binding and is below the efficient wage level where the labor supply curve intersects the marginal product curve. In this case, firms pay the minimum wage, and the markdown is the ratio between the minimum wage and the marginal product, with the level of employment given by the labor supply curve evaluated at the minimum wage (see Figure A.2).

$$
\begin{equation*}
w_{i j o}^{*}=\underline{w}, \quad \mu_{i j o}=\frac{\underline{w}}{\left.\frac{\partial y(z, \ell)}{\partial n_{i j o}}\right|_{n_{i j o}^{*}}}, \quad n_{i j o}^{*}=\left(\frac{\underline{w}}{\mathbf{w}_{j o}}\right)^{\eta_{o}}\left(\frac{\mathbf{w}_{j o}}{\mathbf{W}_{o}}\right)^{\theta_{o}} \mathbf{N}_{o} . \tag{15}
\end{equation*}
$$

In this region, firms pay higher wages and hire more workers than they had done without the minimum wage.

Case III: The minimum wage is binding, and labor supply excess labor demand. The minimum wage is binding and is above the efficient wage level where the labor supply curve intersects the marginal product curve. In this case, firms pay a wage that is equal to both the minimum wage and marginal product, with the employment level given by the marginal product, and firms face an excess of labor supply (see Figure A.3).

$$
\begin{equation*}
w_{i j o}^{*}=\underline{w}=\left.\frac{\partial y(z, \ell)}{\partial n_{i j o}}\right|_{n_{i j o}^{*}}, \quad \mu_{i j o}=1, \quad n_{i j o}^{*}<\left(\frac{\underline{w}}{\mathbf{w}_{j o}}\right)^{\eta_{o}}\left(\frac{\mathbf{w}_{j o}}{\mathbf{W}_{o}}\right)^{\theta_{o}} \mathbf{N}_{o} . \tag{16}
\end{equation*}
$$

In this region, firms pay higher wages and hire fewer workers than they had done without the minimum wage.

Equilibrium. Given a minimum wage $\underline{w}$, the general equilibrium of this economy is a set of organizational structures $\left\{\ell_{i j}^{*}\right\}$, aggregate disutilities of labor supply $\left(N_{w}^{*}, N_{m}^{*}\right)$, and employment levels $\left\{n_{i j w}^{*}, n_{i j m}^{*}\right\}$ such that:

1. Labor supply: Households choose aggregate disutility $N_{o}^{*}$ and labor supply to each firm $\left\{n_{i j o}^{*}\right\}$ to maximize utility. That is, Equation (6) and Equation (7) hold $\forall o \in\{w, m\}$.
2. Firm organization: Firms optimally choose the organizational structure: $\ell_{i j}^{*}$. That is, Equation (9) holds $\forall j \in[0,1], \forall i=\left\{1, \ldots, M_{j}\right\}$.
3. Labor Demand: Firms optimally choose employment $\left(n_{i j w}^{*}, n_{i j m}^{*}\right)$. That is, Equations (12)-(16) hold $\forall j \in[0,1], \forall i=\left\{1, \ldots, M_{j}\right\}$.
4. Market Clearing: Labor supply and demand are given by Equations (12) and (15) for firms in Cases I and II. For firms in Case III, households supply the labor demand $n_{i j o}^{*}$ given by Equation (16).

Note that the equilibrium considers market clearing in the presence of minimum wages. To handle non-market-clearing wages, we solve the equilibrium using a shadow wages approach as in Berger et al. (2023b). This approach considers that households perceive a lower wage than the minimum wage for firms in Case III, which implies that the excess labor supply at the minimum wage is reallocated towards other firms (see Appendix D).

### 3.2 Market Characterization

Before quantifying model parameters, we discuss the interaction between firm organization, market concentration, and wage markdowns in equilibrium. Figure 3 plots the average level of employment, log wages, and wage markdowns across firms of distinct productivity under the benchmark quantification of parameters.

Consider first the top left panel, which displays the employment level of production workers (blue) and managers (green) across firms. All firms hire production workers, as they are essential for production. However, only relatively high-productivity firms find it optimal to hire managers. These firms have more incentives to produce at large scales, and hiring managers enable them to manage a larger workforce. As a result, they use managerial delegation to dampen the diminishing marginal returns to the labor of production workers. The top right panel shows the wage level across firms for each occupation. Production workers earn wages close to the minimum wage when they work at low-productivity firms. Furthermore, managers earn substantially higher wages than production workers. This occurs because the

Figure 3: Oligopsonistic Market with Firm Organization


Note: Figures constructed from the model under the estimated parameters in Table 3.
aggregate disutility from labor supply is higher for managers and because the ratio of production workers to managers is high, which drives up the marginal productivity of managers relative to that of production workers. Moreover, the model captures the fact that wage dispersion between both occupations increases with firm size. This result stems from the fact that the ratio of production workers to managers increases with firm size and, therefore, and so does the ratio of their marginal productivity.

Lastly, the bottom panel of Figure 3 plots wage markdowns across firms as the ratio between wages and the marginal product of labor for each occupation. Overall, wage markdowns are wider for managers than for production workers for three reasons. First, minimum wages are more likely to be binding for production workers. Second, managers have a lower degree of firm-substitutability. If firms had the same payroll share in both markets, they would exert a wider markdown over managerial wages because they would internalize that managers bear higher across-firm mobility costs, which makes them less responsive to labor demand changes. Third, market payroll concentration is also higher for managers. If both occupations had the same degree of firm-substitutability, firms would exert a wider markdown over managerial wages because the labor supply elasticity of managers that firms face would be closer to the across-market elasticity than that of production workers.

## 4 Quantification of the Model

The quantification of the model parameters proceeds in three steps. First, we exogenously calibrate the aggregate Frisch elasticity of labor supply. Second, we calibrate the withinmarket elasticity parameters from the correlation between firms' wages and employment, and we use an indirect inference approach to estimate the across-market elasticity from plausibly exogenous changes in municipality's labor demand. Third, we estimate the remaining model parameters using the Simulated Method of Moments (SMM) approach. The quantification of the parameters targets moments of the firm organization, such as the average firm size or span of control. Together, these moments determine the characteristics of the average firm across markets. Then, we target observed market differences in wages, employment, number of firms, and payroll concentration. Finally, we compare the model simulated moments and their data counterparts to assess the model predictions, and we exploit mass-layoff shocks to validate the performance of the model equilibrium responses.

### 4.1 Estimation

Table 3 summarizes the model parameters. We calibrate outside the model the aggregate Frisch elasticity. We follow Berger et al. (2022) by setting $\gamma=0.5$, which is within the range that the Congressional Budget Office considers for policy evaluation. We endoge-
nously calibrate the within-market elasticities to match the correlation between firm wages and employment after controlling for municipality-year fixed effects. Moreover, we adopt an indirect inference approach to estimate the across-market elasticities from plausibly exogenous labor demand changes at the municipality level, which we generate with a Bartik-type instrument that exploits national sector employment trends interacted with baseline exposure shares during the Great Recession. Then, we estimate the remaining parameters by the SMM approach. In particular, we set the parameter values to minimize the percentage difference with equal weighting between the vector of model moments and its data counterpart. Appendix E. 1 shows the fit of our targeted parameters. Next, we describe each parameter and its most informative moment in detail.

Labor disutility shifter $\left(\phi_{w}, \phi_{m}\right)$. The moment most closely associated with the labor disutility shifter of production workers $\phi_{w}$ is the average firm size in terms of production workers. In the data, the average firm hires 5.3 production workers. For the labor disutility shifter of managers $\phi_{m}$, we include as the most linked moment that about 19 percent of all employees are managers.

Span of control $(\alpha)$. The ratio of production workers to managers partially depends on the span of control parameter. When this parameter increases, the technology converges towards a production function with constant marginal returns to labor, thus decreasing the incentives for managerial delegation. The closest moment we use for this parameter is that the median multi-layer firm has 3.1 workers per manager.

Efficiency of labor $\left(\varphi_{w}, \varphi_{m}\right)$. The efficiency of labor parameters is informative of wages. Thus, we include as targets the mean monthly wage of $717 €$ for production workers and the wage gap in mean wages between managers and production workers, which is equal to 0.73 log points.

Dispersion in firm productivity $\left(\sigma_{z}\right)$. The most associated moment to the standard deviation of firm productivity is the employment-weighted average HHI across local labor markets of production workers, which equals 0.19.

Market Amenities ( $B_{i j w}$ ). Regarding market amenities, we note that only 12 percent of production workers belong to markets with less than ten firms, despite these markets

Table 3: Parameterization

| Parameter Value | Description | Value | Moment |
| :--- | :--- | :--- | :--- |
| Panel I: Exogenous Calibration |  |  |  |
| $\gamma$ | Aggregate Frisch elasticity | 0.50 | Berger et al. (2022) |
| Panel II: SMM Estimation |  |  |  |
| A: Preferences |  |  |  |
| $\phi_{w}$ | Labor disutility shifter: workers | $1.67 \times 10^{-4}$ | Average firm size |
| $\phi_{m}$ | Labor disutility shifter: managers | 0.13 | Share managers |
| B: Firm Organization | Span of control |  |  |
| $\alpha$ | Worker efficiency | 0.76 | Median span of control |
| $\varphi_{w}$ | Managerial efficiency | 1.356 | Mean wage of prod. workers |
| $\varphi_{m} / \varphi_{w}$ | Std. Dev. firm TFP | 1.11 | Wage gap managers and prod. workers |
| $\sigma_{z}$ |  |  |  |
| $C:$ Market Characteristics | Amenities in small markets | 0.46 | Share workers in markets $M_{j} \leq 10$ |
| $B_{i j w}$ |  |  |  |
| Panel III: Endogenous Calibration |  | Across-market firm substitutability | $(1.52,0.95)$ |
| $\left(\theta_{w}, \theta_{m}\right)$ | Across-municipality labor supply elasticity |  |  |
| $\left(\eta_{w}, \eta_{m}\right)$ | Within-market firm substitutability | $(19.97,6.10)$ | Within-market labor supply elasticity |

Source: The Table reports the quantification of model parameters. Panel I reports the parameters that we calibrate outside the model. Panel II reports the estimated parameters using the SMM approach.
representing nearly 65 percent of the total. We set a common market amenity for production workers in markets with less than ten firms. Then, our approach includes the share of production workers in those markets as the most related moment. The rationale for using amenities rather than productivity differences is that we would require too low productivity in small markets, which would highly overestimate the share of minimum wage earners in such places.

Firm Distribution ( $G$ ). The distribution of the number of firms across markets, $M_{j} \sim$ $G(\cdot)$, combines a discrete mass at $m_{j}=1$ with a Pareto distribution. To estimate these parameters, the most associated targeted moments include that 29 percent of markets have just one firm, the average market has 17.4 firms, and the standard deviation in the number of firms equals 59.9.

Across-market substitutability $\left(\theta_{w}, \theta_{m}\right)$. The across-market firm substitutability parameters govern how greater market productivity translates into more employment. When firm substitutability is high, employment in a particular market is highly responsive to increased market productivity. We use an indirect-inference approach for each occupation to match the reduced-form inverse labor supply elasticity from a municipality-level regression. We estimate the following equation:

$$
\begin{equation*}
\log w_{m, o, t}=\gamma \log L_{m, o, t}+\alpha_{m, o}+e_{m, o, t}, \tag{17}
\end{equation*}
$$

where $w_{m, o, t}$ is the mean wage in municipality $m$ for occupation $o$ in period $t, L_{m, o, t}$ is total employment in that municipality, and $\alpha_{m, o}$ are municipality fixed effects. The main threat to identification is that employment and wages in a municipality may vary over time due to changes in labor supply. To overcome this problem, we use a shift-share instrument for employment (Blanchard et al., 1992): ${ }^{9}$

$$
\begin{equation*}
\hat{L}_{m, o, t}=\sum_{s}(\underbrace{\frac{L_{i, m, s, o, 2007}}{\sum_{i} L_{i, m, s, o, 2007}}}_{\text {Industry-Municipality Share }} \times \underbrace{\sum_{i} L_{i, s, o, t}}_{\text {National Employment in Sector } s}) . \tag{18}
\end{equation*}
$$

This instrument predicts employment in each municipality as the exposure-weighted average

[^7]of national sector employment, where the weights are the employment shares of each sector in the municipality in the initial period. With this approach, we estimate the coefficient $\gamma$ from within-municipality, across-time variation in wages and employment that arises from national sector employment shocks and the municipality's initial exposure to them, which plausibly represent exogenous changes in the municipality's labor demand. We also restrict the sample to municipalities with a mean wage higher than 25 percent of the minimum wage in the reference year. We impose this restriction to exclude municipalities where a high share of firms pay wages close to the minimum wage, as the labor supply elasticity of these firms is not informative of the across-market elasticity because they may pay the minimum wage either before or after the labor demand shock. Table E. 2 reports the IV results for each occupation. The implied coefficients are 1.5 for production workers and 0.92 for managers.

To replicate this regression in the model, we randomly assign markets to each municipality to approximate the number of markets that the average municipality has in the data while keeping a reasonable sample size of municipalities. Then, we simulate two periods, where the second period involves random productivity shocks at the municipality level from a standard log-normal distribution with $\sigma=0.05$. Moreover, we restrict the sample of municipalities to those with a mean wage higher than 25 percent of the model minimum wage in both periods. Finally, we choose $\left(\theta_{w}, \theta_{m}\right)$ to target the inverse labor supply elasticity $\gamma$ that results from estimating the regression in Equation (17) with the simulated sample. We infer an across-market elasticity of 1.5 for production workers and 0.95 for managers.

Within-market substitutability $\left(\eta_{w}, \eta_{m}\right)$. Lastly, we calibrate the within-market elasticity parameters, which are informative of the relationship between firms' wages and employment for the sub-sample of unconstrained firms in each market. In particular, the inverse labor supply curve in Equation (7) delivers the following equilibrium relationship between (log) wages and (log) employment:

$$
\log \left(w_{i j o}^{*}\right)=\frac{1}{\eta_{o}} \log \left(n_{i j o}^{*}\right)+\underbrace{\left(\frac{1}{\theta_{o}}-\frac{1}{\eta_{o}}\right) \log \left(\mathbf{n}_{\mathbf{j o}}\left(n_{i j o}^{*}, n_{-i j o}^{*}\right)\right)}_{\text {Effect of payroll share on wages stems from } \mathbf{n}_{\mathbf{j}}}+\frac{1}{\theta_{o}} \log \left(\mathbf{N}_{\mathbf{o}}\right)+\log \left(\mathbf{W}_{\mathbf{o}}\right) .
$$

Note that, conditional on common market features, all firms face the same labor supply elasticity $\eta_{o}$ for each occupation. This occurs because the effect of the payroll share on
the labor supply elasticity shuts down when we control for market employment. We use this insight to obtain a theory-consistent estimate of the within-market elasticity for each occupation. In particular, the previous equation implies the following empirical reduced-form relationship for the inverse labor supply curve:

$$
\begin{equation*}
\log \left(w_{i j o, t}\right)=\beta_{o} \log \left(n_{i j o, t}\right)+\mu_{j o, t}+\nu_{i j o, t}, \tag{19}
\end{equation*}
$$

where $\mu_{j o, t}$ stands for the market-year fixed effects that empirically control for common labor demand and supply shocks across firms in the same market year. Our coefficient of interest is $\beta_{o}$. In the model, conditional on the sub-sample of unconstrained firms, the OLS regression of Equation (19) separately for each occupation identifies the within-market elasticity as $\hat{\eta}_{o}=1 / \hat{\beta}_{o}$. This regression exploits the cross-sectional variation in employment and wages that uniquely stems from labor demand differences across firms in the same market while keeping their labor supply curve fixed. The intuition is as follows. Firms pay different wages and hire a different number of workers because they are heterogeneous in productivity. Increasing the productivity of a firm has two equilibrium effects. First, the labor demand curve shifts to the right because the marginal productivity rises. Second, the labor supply curve shifts to the right because the strategic complementarities from Cournot's competition imply that competitors restrict employment. The coefficient $\beta_{o}$ absorbs the first effect while market-fixed effects absorb the second effect.

In our baseline specification, we separately estimate Equation (19) by OLS for each occupation. Regarding the sample selection, we restrict to firms paying wages at least one percent higher than the minimum wage each year. This regression implies a within-market labor supply elasticity of 19.9 for production workers and 6.1 for managers (see Table E.3). In addition, since the error may capture firm-specific labor supply considerations that threat identification, we also provide the results from an IV regression that uses a traditional Bartik instrument to predict firms' employment from national sector employment trends and initial shares (see Appendix E.2). This alternative specification provides similar results. Overall, we estimate that production workers are three times as responsive to changes in labor demand as managers.

### 4.2 Discussion on the Estimation of Labor Supply Elasticities

Given the importance of the firm-substitutability parameters in estimating wage markdowns, we provide additional evidence on mobility measures that support our findings and compare our estimates with the literature.

The main finding from our estimates is that production workers are significantly more responsive to exogenous labor demand changes than managers. This result is consistent with the empirical evidence of monopsony power over different worker types. Bachmann et al. (2022) find that the labor supply elasticity of workers performing non-routine cognitive tasks is lower than that of workers performing routine or non-routine tasks. Langella and Manning (2021) show that the quit rate is less responsive to wage changes at the top of the wage distribution. In addition, since the labor supply elasticity of production workers is higher, one would expect that they are also more mobile across markets and firms. Tables (A.4)(A.5) report that production workers are more likely to change municipality and sector than managers. Production workers may be more mobile because they are younger (Molloy et al., 2011; Kennan and Walker, 2011; Faberman et al., 2022) and more likely to have temporary contracts (Kahn, 2012). The results from Tables (A.4)-(A.5) show that both factors largely contribute to the gap in the migration probability.

Comparing our estimates with the literature, we focus on the estimates from three papers that also estimate the firm-substitutability parameters using a general equilibrium model of oligopsony. Berger et al. (2022) estimate the firm-substitutability parameters using an indirect inference approach to replicate size-dependent labor supply elasticities at the firm level, which they estimate using U.S. microdata and exploiting changes in state corporate taxes. Abstracting from different occupations, they define the local labor market as the combination of a three-digit industry and a commuting zone. They estimate $\theta=0.4$ and $\eta=10.9$. Shubhdeep et al. (2023) also estimates the firm substitutability parameters from U.S. microdata and exploits changes in state corporate taxes. However, they estimate skill-specific parameters and use a stochastic market definition that is a subset of 6 -digit industries. For low-skilled workers, they estimate $\theta_{L}=1.9$ and $\eta_{L}=2.4$, and for high-skilled workers, they find $\theta_{H}=2$ and $\eta_{H}=2.5$. Using French data, Azkarate-Askasua and Zerecero (2023) define a local labor market as the intersection between a commuting zone, three-digit industry,
and occupation. Nonetheless, they assume the firm-substitutability parameters to be common across occupations. They find an across-market elasticity $\theta=0.4$ and industry-specific within-market elasticities that range within $\eta \in\{1.2,4.1\}$. Despite using a different methodology, we find our estimates reasonably close to their results. Our estimates of the withinmarket elasticities are within the range of their results for managers but somewhat higher for production workers. We attribute this to Portuguese-specific factors such as the high rate of temporary contracts. Our estimates of the across-market elasticities are also within the range of their results. They are higher than Berger et al. (2022) and Azkarate-Askasua and Zerecero (2023), which is consistent with Portuguese municipalities being smaller than U.S. and French commuting zones. ${ }^{10}$

### 4.3 Model Fit

Before turning to the counterfactual analysis, we discuss how the model captures untargeted moments of employment, wages, firm organization, and market concentration. In addition, we exploit large mass-layoff shocks to assess the model's generated employment and wage responses relative to the ones in the data.

Moments for the minimum wage. We start analyzing the wage distribution, which is key to quantifying wage dispersion and the repercussions of minimum wage policies. Figure 4 displays the occupation wage distribution in the model and data. The model fits most of the wage distribution for both occupations. Importantly, it replicates the wage distribution at the bottom deciles. This is key because minimum wage policies primarily affect workers whose wage is close to the statutory minimum wage. Since we assume that workers are homogeneous in talent within occupations, the model underestimates wages in the upper deciles. However, minimum wage policies have little impact on these workers.

Panel A in Table 4 displays how the model fits moments related to the share of workers whose wages are close to the minimum wage. The model somewhat overvalues the share of minimum wage earners for both occupations, but it reflects that managers are significantly

[^8]Figure 4: Wage Distribution across Occupations


Note: The Figures show the wage distribution in the model and data for production workers (left) and managers (right).
less likely to earn the minimum wage than production workers. Moreover, the model gets right the share of workers in each occupation conditional on being a minimum wage earner. That is, the model matches that more than 90 percent of employees are production workers. Furthermore, the model matches that most production workers earn less than 30 percent of the minimum wage $(700 €)$, whereas this proportion falls to around 15 percent for managers. Hence, the model captures that minimum wage policies mainly affect production workers.

Moments of firm organization. The distribution of firm wages, the degree of firm substitutability, and local amenities determine the firm size distribution in the model. This distribution is essential to understanding the misallocation of labor across firms that results from high-productivity firms exerting wider markdowns. Panel B in Table 4 shows that the distribution in the model is somewhat similar to that of the data for production workers. Most firms are small and hire less than two employees, whereas a few firms are relatively large and hire more than 60 workers.

The share of multi-layer firms is key for explaining the differences in market concentration in the markets of production workers and managers, as it determines the relative number of

Table 4: Untargeted Moments

|  | Production Workers | Managers |  |  |
| :--- | :--- | :--- | :--- | :--- |
|  | Model | Data | Model | Data |
| Panel A: Minimum Wage |  |  |  |  |
| Share minimum wage earners | 0.31 | 0.22 | 0.12 | 0.04 |
| Share \| Minimum wage earner | 0.91 | 0.96 | 0.09 | 0.04 |
| Share wage $\leq 700$ | 0.55 | 0.69 | 0.15 | 0.12 |
| Panel B: Firm Organization |  |  |  |  |
| Share multi-layer | 0.33 | 0.33 |  |  |
| Share workers in multi-layer | 0.36 | 0.43 |  |  |
| Share workers in multi-layer $\mid \mathrm{HHI}_{j} \leq 0.20$ | 0.31 | 0.37 |  |  |
| Share workers in multi-layer $\mid \mathrm{HHI}_{j}>0.20$ | 0.49 | 0.55 |  |  |
| P25 firm size | 1 | 1 | 2 | 1 |
| P50 firm size | 2 | 2 | 3 | 1 |
| P90 firm size | 14 | 9 | 9 | 5 |
| P99 firm size | 52 | 59 | 15 | 34 |
| Panel C: Market Concentration |  |  |  |  |
| Weighted mean HHI |  |  |  |  |
| Weighted mean Max sij |  |  |  |  |

Note: The Table reports untargeted moments of the distributions of wages, firm organization, and market payroll concentration. For each occupation, we report the statistics from the data and baseline model.

Figure 5: Distribution of Employment across Markets


Note: The Figures show the cumulative fraction of employment across local labor markets ranked by their level of concentration in the model and data for production workers (left) and managers (right).
competing firms across these markets. Panel C shows that one-third of firms hire managers in both the model and data. Moreover, these firms hire nearly 40 percent of production workers, which is also relevant because multi-layer firms are on average bigger and increase market concentration. The model can also explain our motivational fact relating the employment share of multi-layer firms to market concentration. In particular, the share of production workers employed in multi-layer firms is about 20 pp higher in relatively high-concentrated local labor markets. The model rationalizes this fact because the few firms that usually compete in these markets can attract many production workers and managers at relatively low wages due to the low degree of substitutability across markets. As a result, they produce at large scales and find managerial delegation profitable.

Moments of market concentration. Next, consider the distribution of employment across markets that differ in the level of payroll concentration, which is endogenous in the model due to the agents' labor demand and supply decisions. This distribution is fundamental to measuring the wage markdown of the average employee. Figure 5 shows that the model closely matches the distribution of employment across markets that differ in terms of the HHI.

Two features stand out. First, the model rationalizes that most workers sort into a handful of relatively low-concentrated labor markets. These markets attract most workers because they have many firms that pay higher wages due to the relatively high degree of competition. These markets are also more attractive for production workers because they have relatively higher local amenities. Second, managers are more likely to belong to a market with higher levels of concentration relative to production workers. This heterogeneity occurs because there is a higher proportion of managerial markets with higher concentration levels, and managers face higher across-market mobility costs, discouraging them from leaving markets that pay relatively low wages.

Lastly, consider how the model fits two statistics of market concentration. Panel C in Table 4 shows that the model replicates that the average manager works in a market with an HHI of around one-third; the same one would observe with three equally sized firms. In addition to aggregate concentration measures, the model establishes a direct relationship between a wage markdown and the firm's market payroll share. As a result, the model needs to predict realistic payroll shares for the biggest firms. The last row in Panel C of Table 4 shows that the model captures that the average production worker works for a firm with nearly one-third of the market payroll, whereas the average manager works for a firm with about 40 percent of the market payroll.

Simulating mass-layoff shocks. Mass-layoff shocks are relevant to assess the performance of employment and wage equilibrium responses in the model. The model incorporates two main channels through which areas adjust to this shock: (i) outflows to non-employment and (ii) worker reallocation to other firms, either in the same or a different area. The empirical evidence suggests that most of the impact of mass-layoff shocks on regional employment depends on the degree of worker reallocation across regions (Foote et al., 2019; Gathmann et al., 2020). ${ }^{11}$ Therefore, we use this simulation as a validation exercise of the across- and within-market firm substitutability estimates, which are the key drivers of the allocation of workers across and within markets.

[^9]Table 5: Estimates of Mass Layoff Shocks on Municipalities

|  | Production Workers |  | Managers |  |
| :--- | :---: | :---: | :---: | :---: |
|  | Model | Data | Model | Data |
| Panel A: Log Wages |  |  |  |  |
| Event Region | $-0.006^{* * *}$ |  |  |  |
|  | $(0.001)$ | -0.007 | $-0.012^{* * *}$ | -0.006 |
| Panel B: Log Employment |  |  |  |  |
| Event Region | $-0.017^{* * *}$ | $-0.061^{* * *}$ | $-0.011^{* *}$ | $-0.040^{* *}$ |
| Observations | $20.001)$ | $(0.013)$ | $(0.005)$ | $(0.019)$ |

Note: The Table reports the empirical (Data) and simulated (Model) results from exploiting mass-layoff shocks. For the Data, we show the average treatment effect of mass-layoff shocks on average municipality wages (Panel A) and total municipality employment (Panel B). The dependent variable is the $\log$ municipality's employment and wages. Standard errors are in parentheses.

Next, we show the results from this exercise and relegate the details of the methodology to the Appendix E.3. Table 5 reports the empirical and simulated results of the impact of mass-layoff shocks on average municipality wages and total municipality employment. Our empirical results show that municipalities with at least one mass-layoff event experience an average decrease of 6.1 percent in the local employment of production workers and 4 percent in the local employment of managers compared to municipalities without mass layoffs. We do not find a significant impact of mass layoffs on average municipality wages. These results align with Gathmann et al. (2020) for Germany, which find a drop of 3.7 percent in local employment and an insignificant response in wages following a mass layoff.

Regarding the results in the simulated data, the model reproduces that mass layoffs have a small effect on average municipality wages for production workers. For managers, the point estimate is twice more negative in the model, but it is in the range of the confidence interval of the data. In the model, wages decrease for two reasons. First, the shocked firm becomes less productive. Second, the non-affected firms also decrease wages because their employment share increase. At the same time, wages do not fall much due to the downward rigidity
in wages stemming from minimum wages. In terms of employment, the model accounts for nearly one-fourth of the decrease in employment of managers and production workers. Overall, the model explains a substantial portion of the observed employment decline. We attribute the residual to the lack of factors such as agglomeration economies, input-output linkages, or firm exit.

## 5 Implications of Monopsony Power

In this section, we quantify the distribution of wage markdowns for both occupations and the effect of monopsony power on wage dispersion, firm organization, and welfare. We estimate a markdown of 10.4 percent for the average production worker and 23.1 percent for the average manager. Relative to the efficient economy, monopsony power reduces the managerial wage premium by 11.8 percent and increases the share of multi-layer firms by 11.3 percent because it incentivizes less productive firms to delegate tasks to managers. Overall, monopsony power leads to a welfare loss of 5.7 and 23.1 percent for production workers and managers, respectively. Understanding firms' managerial delegation choices is crucial to explain the reallocation of managers towards more productive firms and accounts for 10 to 30 percent of the change in average market concentration in the efficient economy. Moreover, ignoring that monopsony power incentivizes delegation leads to overestimating the welfare loss of production workers and managers by 0.6 and 2.1 pp , respectively.

### 5.1 Measuring Monopsony Power

This section shows the distribution of wage markdowns across firms for production workers and managers. In the benchmark economy, wage markdowns are below one due to imperfect firm substitutability and firm granularity, implying that wages are below the marginal revenue product of labor. This wedge represents the efficiency loss from monopsony power, as workers would earn the entire marginal product in an efficient economy.

Figure 10 displays the distribution of wage markdowns for both occupations. We estimate an employment-weighted markdown of 10.4 percent over the wage of the average production worker and a markdown of 23.1 percent over the wage of the average manager. Therefore,

Figure 6: Distribution of Wage Markdowns


Note: The Figure plots the distribution of wage markdowns ( $\mu_{i j}$ ) across firms for production workers and managers. The wage markdown is the wedge between the wage and the marginal productivity of labor. Dashed lines display the weighted mean of each variable, where the weight of each firm is its employment size.
the interaction between monopsony power and firm organization reduces the wage dispersion between both occupations. There are three reasons for this fact. First, both the upper $\left(\eta_{o}\right)$ and lower bounds $\left(\theta_{o}\right)$ of the structural elasticities are lower for managers. That is, we estimate that managers bear greater mobility costs of moving to a firm in another region or industry and greater mobility costs of changing firms within the same region-industry. Second, firms tend to have higher managers' payroll shares than production workers. Thus, the labor supply elasticity of managers is closer to the across-market elasticity than that of production workers. In other words, managers find it harder to reallocate toward other firms because more of their alternatives are outside of their current local market. Third, minimum wages mainly constrain low-productivity firms from exercising monopsony power. Thus, minimum wages are more likely to limit monopsony power over production workers, who tend to work at low-productivity firms. Relative to the efficient economy, this heterogeneity in wage markdowns implies a higher distortion in the allocation of managers across firms than production workers. For policy evaluation, this suggests that additional policies targeting the reduction of monopsony power among low-wage workers, such as an increase in the minimum wage, may not effectively mitigate the welfare losses from monopsony power.

### 5.2 Welfare and Efficiency Losses from Monopsony Power

We now study the effect of monopsony power on firm organization, wage dispersion, efficiency, and welfare. Moreover, we estimate the extent to which the impact of monopsony power on these outcomes is attributable to the endogenous organizational choices of firms. We compute the efficient allocation by setting wage markdowns to one, i.e., equalizing wages to the marginal product of labor for all firms. The left column in Table 6 summarizes the results by comparing the aggregate outcomes in the efficient relative to the benchmark economy.

Panel A in Table 6 shows the change in employment across occupations. When firms have monopsony power, they internalize that a higher level of employment also involves higher wages, i.e., they internalize an upward-sloping labor supply curve. Then, firms find it optimal to restrict employment relative to the efficient allocation to reduce labor costs. We find that employment would rise by 5.6 and 9.3 percent for production workers and managers in an efficient economy, respectively. Managers' employment response is higher because firms exert wider markdowns over their wages. Besides changes in aggregate employment, monopsony power also distorts labor allocation across firms because high-productivity firms restrict employment the most, as they set the widest markdowns. The top panels in Figure 7 show the reallocation of employees from less to more productive firms. We observe that firms in the top decile of productivity raise the number of production workers and managers by about 12 and 21 percent, respectively. The increase in employment and the reallocation of workers leads to an increase of 3.3 percent in aggregate output.

Regarding the impact of monopsony power on wages, the top panels in Figure 8 show how mean wages change across the firm productivity distribution. In an efficient economy, wages especially increase at high-productivity firms, which exert the widest markdowns. As a result, wage dispersion within occupations also increases. Since managers bear wider markdowns, the wage increase is greater for managers. In particular, Panel B shows that the mean wage of production workers and managers increase by 10 and 23 percent, respectively. As a result, dispersion in mean wages across occupations rises by 11.8 percent.

To understand why managerial employment and wages drop at middle-productivity firms, we need to analyze the effect of monopsony power on the internal organization of firms. Panel C shows, in line with the empirical evidence, that more firms adopt a multi-layer structure

Table 6: Change in Counterfactual Relative to Benchmark

|  | Efficient Economy | Minimum Wage Reform |
| :--- | :---: | :---: |
| Panel A: Employment \& Output | \% Change | \% Change |
| Production workers | 5.57 | -1.28 |
| Managers | 9.29 | -0.99 |
| Aggregate Employment | 6.33 | -1.21 |
| Output | 3.25 | -0.04 |
| Panel B: Wages | 10.04 |  |
| Mean: Production workers | 23.02 | 2.06 |
| Mean: Managers | 11.79 | 0.94 |
| Ratio |  | -0.87 |
| Panel C: Firm Organization | -11.27 | -0.56 |
| Share multi-layer firms | -3.64 | -0.48 |
| Median span of control | 5.00 | 0.63 |
| Mean HHI: Production Workers | 7.95 | 0.07 |
| Mean HHI: Managers | 23.13 | -0.72 |
| Panel D: Welfare |  |  |
| Welfare: Production workers |  |  |
| Welfare: Managers |  |  |

Note: The Table reports the percent change in aggregate outcomes in the counterfactual relative to the benchmark economy. In the first column, the counterfactual consists of an efficient economy where wage markdowns equal one. In the second column, the counterfactual consists of an economy with a ten percent higher minimum wage than the benchmark economy.

Figure 7: Effect of Monopsony Power on Employment Reallocation


Note: The top panels in the Figure plot the percent change in employment of production workers (left) and managers (right) across firms in the efficient relative to the benchmark economy. The bottom panel shows the reallocation of managers in an efficient economy where firms exogenously keep the organizational structure of the benchmark economy. Both counterfactual simulations represent an efficient economy where wages are equal to the marginal product of labor. We classify firms into ten bins according to their productivity, where a higher bin implies higher productivity.

Figure 8: Effect of Monopsony Power on Wages


Note: The top panels of the Figure plot the percent change in mean wages relative to the benchmark for production workers (left top) and managers (right top). The bottom panel shows the change in mean wages of managers in an efficient economy where firms exogenously keep the organizational structure of the benchmark economy. Both counterfactual simulations represent an efficient economy where wages are equal to the marginal product of labor. We classify firms into ten bins according to their productivity, where a higher bin implies higher productivity.

Figure 9: Share of Multi-layer Firms in the Efficient Relative to Benchmark Economy


Note: The Figure plots the percent change in the share of multi-layer firms, where we express the change as a fraction over the total number of firms (right) and over the total number of firms in the same productivity bin (left). The counterfactual simulation represents an efficient economy where wages are equal to the marginal product of labor. We classify firms into ten bins according to their productivity, where a higher bin implies higher productivity.
when they have monopsony power. In particular, we estimate that the proportion of firms that delegate to managers falls by 11.3 percent, from 33 to 29 percent of all firms. Furthermore, the share of multi-layer firms especially decreases at medium-productivity firms, as we observe in Figure 9. The reason is that monopsony power enables medium-productivity firms to attract employees at relatively low wages, incentivizing them to be multi-layer organizations. This result implies that managers at medium-productivity firms experience the most substantial decline in employment and even a decrease in wages. Notably, when we get rid of the effect of multi-layer firm exit on these outcomes, i.e., we analyze what happens to employment and wages in a counterfactual efficient economy where firms exogenously choose the benchmark organizational structure, we observe that managers at middle-productivity firms would get employment and wage gains due to the reallocation of production workers (see the bottom panels of Figure 7 and Figure 8). Regarding managers' span of control, we estimate that it decreases by almost 3.6 percent because firms increase their managerial workforce. We also find that the average HHI in the market of production workers and man-
agers increases by 5 and 7.9 percent, respectively, due to the reallocation of labor towards high-productivity firms and the decrease in the number of firms that hire managers.

Panel D reports the welfare losses from monopsony power. The welfare gain in the efficient economy is substantially higher than the output gain because there is a redistribution channel that increases labor income at the expense of profits in addition to the efficiency gain. Measuring welfare as the consumption equivalent that would make households indifferent between the efficient and benchmark economy, we estimate that welfare increases by 5.7 and 23.1 percent for production workers and managers, respectively.

### 5.3 Firm organization channel

Our structural model allows us to quantify the extent to which changes in economic outcomes in the efficient economy stem from the endogenous organizational choices of firms. To isolate this channel, we simulate a counterfactual efficient economy where firms exogenously keep the same number of layers they choose in the benchmark economy. Then, we compute the change in each economic outcome between this counterfactual and the benchmark economy. This change is attributable to all the mechanisms included in the model except for the firm organization channel. Hence, we attribute to firm organizational choices the difference in the changes between the efficient counterfactual economy, where the layer structure is endogenous, and the efficient counterfactual economy, where this choice is exogenous. Figure 10 shows how much firm organization accounts for changes in the market structure and welfare of production workers and managers. Two results stand out.

First, the firm organization channel explains about 30 percent of the change in the average concentration level of managerial markets. In particular, the average HHI of managerial markets increases by 7.7 percent in the efficient economy. Out of this increase, 2.3 pp are attributable to delegation decisions. The reason is that the number of multi-layer firms decreases when firm organization is endogenous, further contributing to increased market concentration because fewer firms participate in managerial markets. Moreover, the firm organization channel also explains nearly 10 percent of the change in the average concentration level in production worker markets. This result stems from the technological reciprocity between worker types, which implies that organizational decisions also affect the distribution

Figure 10: Firm Organization Channel


Note: The Figures display how much the firm organization channel, i.e., the endogenous firms' choice of layers, accounts for the percent change between the efficient and benchmark economies in several outcomes of production workers (left) and managers (right). For instance, the endogenous organizational choice of firms explains about 30 percent of the change in the average level of payroll concentration in managerial markets between the efficient and benchmark economies.
of production workers across firms.

Second, ignoring firm organization contributes to overestimating the welfare gains of both worker types in the efficient economy by about 9.1 percent. Regarding the welfare of managers, we overestimate it by about 2.1 pp when the organization of firms is exogenous ( $25.2 \%$ ) relative to endogenous ( $23.1 \%$ ). That is, the endogenous organizational response of firms in an efficient economy reduces the welfare of managers. Since fewer firms adopt a multi-layer structure in an efficient economy, the demand for managers falls, and so does their consumption level. Moreover, their disutility of labor decreases because their aggregate labor supply falls. Regarding the welfare of production workers, we overestimate it by 0.6 percentage points when the organization of firms is exogenous (6.3\%) relative to endogenous (5.7\%). In this case, almost the entire negative effect of firm organization on production workers' welfare stems from an increase in their disutility of labor supply, as the decrease in the share of multi-layer firms also induces the reallocation of some production workers.

Overall, we show the impact of monopsony power on firm organization, efficiency, and welfare in the Portuguese economy. Regarding the importance of the firm organization channel, we emphasize that it is informative of the impact of monopsony power on employment at middleproductivity firms, welfare, and the market structure. Regarding the policies that aim to reduce the welfare losses from monopsony power, our results point out that the success of these policies especially relies on their ability to reduce the markdowns over managerial wages. In the following section, we assess the effectiveness of Portuguese minimum wage policies in reducing the monopsony power of firms.

## 6 Minimum Wage Policies

Among other reasons, minimum wage policies aim to improve the well-being of low-income workers by reducing the wage-setting power of firms. In this section, we find that recent raises in the statutory Portuguese minimum wage lower overall welfare. Thus, we analyze occupation-based minimum wages as an alternative. Increasing only the minimum wage for managers gives them up to a 0.5 percent welfare gain but slightly decreases production workers' welfare. We also compute the optimal combination of occupation-based minimum wages that generates a Pareto improvement relative to the benchmark economy. We find that this policy brings about a welfare gain of 0.3 and 0.2 for production workers and managers, respectively. Thus, this optimal policy recovers 5.3 and 0.9 percent of their welfare losses from monopsony power, respectively.

### 6.1 Minimum Wage Reforms Implemented in Portugal

Raising the minimum wage mitigates monopsony power in a firm by inducing this firm to increase employment and wages as long as there is a wedge between wages and the marginal revenue product of labor. We assess whether the Portuguese reforms that raised the real minimum wage by ten percent between 2016 and 2019 were effective in mitigating monopsony power. ${ }^{12}$ We focus on this period because it involves a meaningful and permanent minimum

[^10]wage increase, and it coincides right after our calibration period and before the COVID-19 crisis. The rightmost column in Table 6 reports the percent change in aggregate outcomes in the minimum wage counterfactual relative to the benchmark economy.

Panel A in Table 6 shows that the increase in the minimum wage reduces aggregate employment by about 1.3 percent. Although employment falls for both occupations, the employment decline is larger for production workers, who are more likely to work at firms that exert narrow markdowns and pay low wages. The negative effect on employment does not generate large efficiency losses, as output almost remains unchanged. The reason is that the minimum wage increase also induces workers to relocate to high-productivity firms, reducing the misallocation of labor. There is a reallocation effect because the rise in the minimum wage leads to employment losses at less productive firms, part of which relocates to more productive firms. Figure 11 shows the reallocation of both worker types towards high-productivity firms. Note that even a slight employment increase at high-productivity firms crowds out most of the employment drop at low-productivity firms, as high-productivity firms account for most of aggregate employment (see Figure A.7).

Regarding the impact of the minimum wage increase on wages, Figure 12 shows that it increases wages at low-productivity firms, which have to pay higher wages. In contrast, when labor relocates towards high-productivity firms, it increases labor supply in those firms, leading to downward pressure on wages. As a result, wage dispersion decreases within occupations. Since the wage change is more intense for production workers, wage dispersion also decreases across occupations. In particular, Panel B shows that the minimum wage reform reduces mean wage dispersion across occupations by 0.9 percent. Lastly, we estimate the own-wage employment elasticity by occupations to find that for production workers, it is -0.6 , and for managers, it is $-1.05 .{ }^{13}$ These elasticities are in the range of other estimates from the minimum wage literature (Dube, 2019).

Next, Panel C reports the effect on firm organization, which is a novel mechanism in our model. We estimate a decrease of almost 0.6 percent in the share of multi-layer firms. Figure 13 shows that this happens mostly at medium productivity bins, even though managers in these firms earn wages well above the baseline minimum wage (see Figure A.8). The higher

[^11]Figure 11: Effect of the Minimum Wage Reform on Employment Reallocation


Note: The Figure plots the percent change in employment of production workers (left) and managers (right) across firms in the minimum wage counterfactual relative to the benchmark economy. In the counterfactual economy, the minimum wage is ten percent higher than in the benchmark. We classify firms into ten bins according to their productivity, where a higher bin implies higher productivity.

Figure 12: Effect of the Minimum Wage Reform on Wages


Note: The Figure plots the percent change in mean wages of production workers (left) and managers (right) across firms in the minimum wage counterfactual relative to the benchmark economy. In the counterfactual economy, the minimum wage is ten percent higher than in the benchmark. We classify firms into ten bins according to their productivity, where a higher bin implies higher productivity.

Figure 13: Effect of the Minimum Wage Reform on the Share of Multi-layer Firms


Note: The Figure plots the percent change in the share of multi-layer firms, where we compare the change over the total number of firms (right) and the total number of firms in the same productivity bin (left).
minimum wage makes their production workers, who earn wages close to the minimum wage, more expensive. Therefore, they have incentives to reduce their workforce and, consequently, the number of layers because a multi-layer organization primarily provides advantages when producing at large scales. The median span of control of managers decreases by 0.5 percent because the cost of production workers relative to managers increases. Regarding the effect on the market structure, the reform induces an increase in market concentration for both occupations due to the reallocation effect and the drop of the managerial layer of firms.

Lastly, consider the effect of the minimum wage increase on welfare in Panel D. It reduces the welfare of production workers by 0.7 percent. Despite a slight increase in consumption, welfare decreases because the reform induces a reallocation of labor across firms that significantly raises their disutility of labor supply. We interpret this last finding as a situation wherein the reallocation of labor across firms involves substantial search costs, increases commuting costs, and diminishes the idiosyncratic taste of production workers toward firms, all in an effort for workers to remain employed. Regarding the effect on managers, raising
the minimum wage decreases the demand for managers in less productive firms, whose share of multi-layer firms drops. Still, it increases the demand for managers in more productive firms due to worker reallocation. Overall, managers' welfare remains constant as both effects cancel out.

### 6.2 Occupation-based Minimum Wage

The heterogeneity in the distribution of wage markdowns across occupations suggests that designing a minimum wage for each occupation could be more effective at tackling the welfare losses from monopsony power. Distinct minimum wages across occupations exist in Australia and many European countries. ${ }^{14}$ Thus, it is natural to ask whether a different minimum wage for managers would improve upon the implemented Portuguese minimum wage, as managers tend to earn wages above the minimum wage and bear wider markdowns than production workers. For this reason, we simulate different scenarios where we set a specific minimum wage for managers while keeping the benchmark minimum wage for production workers. Figure 14 depicts the welfare change of production workers and managers in each counterfactual relative to the benchmark.

We find that the welfare of managers is hump-shaped relative to a manager-specific minimum wage. As this minimum wage increases, the negative employment effects become more predominant and offset the wage gains. We calculate that the welfare gain of managers attains its maximum at 0.5 percent when their minimum wage equals 56 percent of the mean managerial wage $(900 €)$. Regarding production workers, their welfare level is decreasing in the minimum wage of managers mainly due to a consumption drop. Raising the minimum wage of managers diminishes the consumption level of production workers, as firing managers decreases the marginal productivity of the incumbent production workers and incentivizes firms to fire production workers. Hence, only adjusting the manager-specific minimum wage

[^12]Figure 14: Welfare Effect of Occupation-based Minimum Wage


Note: The Figure plots the percent change in the welfare of both managers and production workers in the counterfactual where the minimum wage of managers changes relative to the benchmark economy. We keep the benchmark minimum wage for production workers.
increases welfare inequality between occupations.

Lastly, we compute the optimal combination of occupation-based minimum wages that generate a Pareto improvement relative to the benchmark economy. We find that this combination of occupation-based minimum wages provides a welfare gain relative to the benchmark of 0.3 percent for production workers and 0.2 percent for managers when the minimum wage of production workers is about 63 percent of their mean wage ( $460 €$ ) and the minimum wage of managers is about 50 percent of their mean wage ( $790 €$ ). That is, there is no other combination of occupation-based minimum wages that can make one worker type better off without making the other worker type worse off. Our results show that occupation-minimum wages improve upon uniform minimum wages. However, optimal occupation-based minimum wage policy only mitigates 5.3 and 0.9 percent of the production workers' and managers' welfare loss from monopsony power, respectively.

## 7 Conclusion

Using a model where firms organize production in hierarchies, we show that the heterogeneity in monopsony power between managers and production workers matters for understanding the welfare effects of monopsony power across worker types and the optimal design of minimum wage policies. We estimate that firms exert a 23.1 and 10.6 percent wage markdown over the average manager and production worker, respectively. As a result, managers and production workers experience a 23.1 and 5.7 percent welfare loss from monopsony power, respectively. We also find that monopsony power incentivizes managerial delegation, especially in medium-productivity firms. Thus, ignoring the endogenous delegation choices of firms leads to overestimating these welfare losses by 0.6 and 2.1 pp . Finally, we study the implication for the design of minimum wages to address the welfare losses from monopsony power. We find that moving to an optimal occupation-based minimum wage provides welfare gains for both worker types relative to the benchmark but recovers less than 5 percent of the welfare losses from monopsony power.

We consider two valuable extensions for future research. We believe market concentration and mobility costs may also show a systematic relationship with other market characteristics. For instance, the skill level required in the job, workers' age, or whether the market is formal or not in developing countries. Regarding the model specification, future model extensions may consider worker heterogeneity in productivity and across-occupation mobility.

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## Appendix

## A Appendix: Additional Figures and Tables

Table A.1: Classification of Occupations

| Level | Tasks | Skills |
| :---: | :---: | :---: |
| Top Management | Definition of the firm general policy or consulting on the organization of the firm; strategic planning; creation or adaptation of technical, scientific and administrative methods or processes | Knowledge of management and coordination of firms fundamental activities; knowledge of management and coordination of the fundamental activities in the field to which the individual is assigned and that requires the study and research of high responsibility and technical level problems |
| Middle Management | Organization and adaptation of the guidelines established by the superiors and directly linked with the executive work | Technical and professional qualifications directed to executive, research, and management work |
| Supervisors | Orientation of teams, as directed by the superiors, but requiring the knowledge of action processes | Complete professional qualification with a specialization |
| Higher-skilled Professionals | Tasks requiring a high technical value and defined in general terms by the superiors | Complete professional qualification with a specialization adding to theoretical and applied knowledge |
| Skilled Professionals | Complex or delicate tasks, usually not repetitive, and defined by the superiors | Complete professional qualification implying theoretical and applied knowledge |
| Semi-skilled Professionals | Well defined tasks, mainly manual or mechanical (no intellectual work) with low complexity, usually routine and sometimes repetitive | Professional qualification in a limited field or practical and elementary professional knowledge |
| Non-skilled Professionals | Simple tasks and totally determined | Practical knowledge and easily acquired in a short time |

Sources: (i) Decreto-Lei no․ 121/78 de 2 de Junho, Ministério do Trabalho, (ii) Caliendo et al. (2020).

Table A.2: Share of Occupations

| Level | Share (\%) | Share Hierarchy (\%) | Mean Wage |
| :--- | :---: | :---: | :---: |
| Managers | 19.2 | 100 | 1,696 |
| Top Management | 8 | 41.8 | 2,108 |
| Middle Management | 6 | 31.2 | 1,481 |
| Supervisors and Team Leaders | 5.2 | 27 | 1,308 |
| Workers | 80.8 | 100 | 717 |
| Higher-skilled Professionals | 8 | 9.9 | 1,192 |
| Skilled Professionals | 40.1 | 26.8 | 729 |
| Semi-skilled Professionals | 21.6 | 13.7 | 599 |
| Non-skilled Professionals | 11.1 |  | 562 |

Source: Elaboration based on QP.

Table A.3: Mobility and Sample Characteristics

|  | $(1)$ <br> Production Workers | $(2)$ <br> Managers |
| :--- | :---: | :---: |
| Share Age $\leq 25$ | 0.11 | Mean |
| Share Age $\leq 30$ | 0.25 | 0.04 |
| Share Temporary | 0.31 | 0.17 |
| Share College | 0.07 | 0.16 |
| Share Change Establishment | 0.10 | 0.55 |
| Share Change Municipality | 0.07 | 0.08 |
| Share Change NUTS-3 Region | 0.03 | 0.06 |
| Share Change Sector | 0.06 | 0.02 |
| Observations | $11,286,635$ | 0.05 |

Source: Elaboration based on QP.
Note: All statistics are significantly different at standard statistical levels. We omit standard errors to ease readability.

Table A.4: Occupation and Migration across Municipalities

|  | $(1)$ | $(2)$ | $(3)$ | $(4)$ | $(5)$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Manager | $-0.012^{* * *}$ | $-0.008^{* * *}$ | $-0.012^{* * *}$ | $-0.002^{* * *}$ | $-0.003^{* * *}$ |
| AME/Baseline | $-17.1 \%$ | $-11.4 \%$ | $-17.1 \%$ | $-5.7 \%$ | $-4.3 \%$ |
| Year FE | No | Yes | Yes | Yes | Yes |
| Sex | No | Yes | Yes | Yes | Yes |
| Age | No | Yes | Yes | Yes | Yes |
| Education | No | No | Yes | Yes | Yes |
| Temporary | No | No | No | Yes | Yes |
| Industry | No | No | No | No | Yes |
| N | $6,628,978$ | $6,628,978$ | $6,615,462$ | $6,572,412$ | $6,572,412$ |
| Baseline | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 |

Source: Elaboration based on QP.
Note: The table reports the marginal effects from a Probit regression of inter-municipality migration on a manager dummy. In addition, to better interpret the results, we report the marginal effect relative to the baseline probability of migration of production workers. Each regression column differs in terms of the vector of controls. The sample period ranges from 2010 to 2016. Standard errors are reported in parentheses.

Table A.5: Occupation and Sectoral Mobility

|  | $(1)$ | $(2)$ | $(3)$ | $(4)$ | $(5)$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Manager | $-0.012^{* * *}$ | $-0.008^{* * *}$ | $-0.014^{* * *}$ | $-0.005^{* * *}$ | $-0.006^{* * *}$ |
| AME/Baseline | $-0.0002)$ | $(0.0002)$ | $(0.0002)$ | $(0.0002)$ | $(0.0002)$ |
| Year FE | No | $-13.3 \%$ | $-23.3 \%$ | $-8.3 \%$ | $-10 \%$ |
| Sex | No | Yes | Yes | Yes | Yes |
| Age | No | Yes | Yes | Yes | Yes |
| Education | No | No | Yes | Yes | Yes |
| Temporary | No | No | No | Yes | Yes |
| Industry | No | No | No | No | Yes |
| N | $9,825,202$ | $9,825,202$ | $9,805,652$ | $9,743,686$ | $9,743,686$ |
| Baseline | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 |

Source: Elaboration based on QP.
Note: The table reports the marginal effects from a Probit regression of sectoral mobility (2-Digit) on a manager dummy. In addition, for a better interpretation of the results, we report the marginal effect relative to the baseline probability of sectoral mobility of production workers. Each regression column differs in terms of the vector of controls. The sample period ranges from 2010 to 2016. Standard errors are reported in parentheses.

Table A.6: Distribution of Number of Establishments across Local Markets $\left(M_{j}\right)$

|  | Mean | P25 | P50 | P75 | P95 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Markets of Managers |  |  |  |  |  |
| No Establishments | 120 | 3 | 11 | 39 | 156 |
| Markets of Production Workers |  |  |  |  |  |
| No Establishments | 227 | 5 | 16 | 65 | 241 |

Source: Elaboration based on QP.
Note: The Table reports the (employment weighted) mean, the 25 th, 50 th, 75 th , and 95 th percentile of the number of establishments across local labor markets by occupation.

Figure A.1: Unbinding Minimum Wage in Partial Equilibrium Analysis


Figure A.2: Binding Minimum Wage on the Labor Supply Curve


Figure A.3: Binding Minimum Wage off the Labor Supply Curve


Figure A.4: Market Concentration and Multi-layer Firms: Unweighted


Source: Elaboration based on QP.
Note: The Figure plots the average share of multi-layer firms across local labor markets that differ in the level of HHI. In particular, we compute the share of multi-layer firms and the HHI for each local labor market of production workers. We split the distribution of the HHI into 20 cells of length 0.05 . In each cell, we take the unweighted mean of the share of multi-layer firms across markets.

Figure A.5: Transition Probabilities


Source: Elaboration based on QP.
Note: The Figures display the transition probabilities of changing sub-occupation. The vertical axis represents the suboccupation before the transition, and the horizontal axis the sub-occupation afterward. The left panel shows the unconditional transition probability, whereas the right panel shows the transition probability conditional on changing firms. The black lines delimit the quadrants of moving across or within the two broad occupation categories (managers and production workers), where the top left and right bottom quadrants represent within-occupation transitions.

Figure A.6: Age Distribution across Occupations


Source: Elaboration based on QP.
Note: The Figure displays the age distribution across occupations.

Figure A.7: Employment Share in the Benchmark Economy


Source: Simulations from the model.
Note: The Figure plots the employment share of production workers (left) and managers (right) across firms in the benchmark economy. We classify firms into ten bins according to their productivity, where a higher bin implies higher productivity.

Figure A.8: Mean Wages in the Benchmark Economy


Source: Simulations from the model.
Note: The Figure plots the employment-weighted mean wage of production workers (left) and managers (right) across firms in the benchmark economy. We classify firms into ten bins according to their productivity, where a higher bin implies higher productivity.

## B Appendix: Data

This section provides a detailed description of the data, the occupation and the market definition, as well as the methodology to measure market concentration using the HerfindahlHirschman Index (HHI).

## B. 1 Quadros de Pessoal

Our primary data source is Quadros de Pessoal (QP), an annual census of private sector employees conducted by the Portuguese Ministry of Employment. This census provides matched employer-employee data on all firms based in Portugal with at least one worker, except those related to public administration and non-market services. The database incorporates unique time-invariant identifiers for each firm, establishment, and worker entering the report, which allows tracking them over time. Our sample covers the period from 2010 to 2016 for all results except for estimating the firm-substitutability parameters, which covers 2002 to 2016 because we require more observations.

The worker-level data contains information on each firm's employees as of the October reference week. The variables include age, occupation, monthly earnings, and hours worked. At the firm level, we have information on the industry, the headquarters location, and all its establishments.

Regarding the sample selection, we exclude workers younger than 18 or older than 64 , those working outside of continental Portugal, and those working in agriculture, forestry, fishing, or mining industries. We also exclude apprentices, workers with missing information on earnings or occupation, and workers with misreported identifiers. Most workers with missing earnings include unpaid family members and owners of the firm. In addition, workers with misreported identifiers (e.g., duplicated) account for about $2 \%$ of the sample. Finally, we drop chief executive officers because their market is not local, which is a core feature of the theory in this paper. This selection results in $3,243,966$ workers and $12,073,646$ worker-year observations.

## B. 2 Market Definition

We classify labor markets based on three observable characteristics of the job: geography, industry, and occupation. This classification stems from the fact that workers are more attached to their current labor market because of imperfect geographical mobility and imperfect substitutability of skills across jobs and sectors (Neal, 1995; Kambourov and Manovskii, 2009; Sullivan, 2010; Monte et al., 2018). In particular, we define two broad occupations, i.e., managers and production workers, and define a local labor market for each occupation as the intersection of the geography (Municipality) and industry (2-digit NACE). This selection results in 13,832 and 11,677 local labor markets for workers and managers, respectively.

We use the municipality or concelho administrative division as the benchmark geographic unit, which splits the country into 278 areas of an average of 320 square kilometers. We use the 2-digit NACE classification of industries as a baseline measure. This includes 78 different economic sectors such as Manufacture of food products or Accommodation and food service activities. Given that our model does not distinguish between across-industry and acrossregion mobility, we use these baseline definitions because worker transitions are similar in both cases. In particular, the unconditional across-municipality and across-industry annual transition probabilities are 9.8 percent.

Regarding the occupational definition, the Portuguese law obliges firms to assign their workers to an occupational category based on tasks performed and skills required so that each category considers the level of the worker within the firm's hierarchy in terms of increasing responsibility and task complexity. We follow a hierarchical classification similar to Caliendo et al. (2020). In particular, we partition professional categories into two layers. We assign top executives, intermediary executives, supervisors, and team leaders to the management layer. In addition, we assign higher-skilled professionals, skilled professionals, skilled professionals, semi-skilled professionals, and non-skilled professionals to the bottom layer. To distinguish between managers and other occupations, the critical difference is that managers are responsible for the organizational policies of the firm and their adaptation, which require a high degree of qualification in terms of direction, guidance, and coordination of the firm

Table B.1: Summary Statistics at the Establishment Level

|  | Mean | P10 | P25 | P50 | P75 | P90 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Production Workers |  |  |  |  |  |  |
| Monthly Wage | 718 | 518 | 588 | 756 | 1,082 | 2,159 |
| Managers |  |  |  |  |  |  |
| Monthly Wage | 1,698 | 937 | 1,346 | 2,059 | 3,065 | 6,441 |
| Span of Control | 8 | 1 | 3 | 8 | 17 | 70 |

Source: Elaboration based on Quadros de Pessoal.
Note: The Table reports the mean, 10th, 25 th, $50 \mathrm{th}, 75 \mathrm{th}$, and 90 th percentile of the individual distribution of wages for managers and non-managers. Wages are base wages (excluding supplementary payments) expressed in full-time equivalent units. In addition, it reports the same distributional moments for the span of control, which we define as the ratio of nonmanagers to managers within an establishment.
fundamental activities. ${ }^{15}$

## B. 3 Summary Statistics

Our classification of occupations implies that 19 percent of workers are managers, while the remaining 81 percent are production workers. Table B. 1 reports summary statistics of the wage distribution for each occupation. Along the distribution, managers earn higher wages than production workers, and this gap particularly widens for high-paid workers. Managers earn around twice as much in the bottom quartile as production workers. In the top quartile, managers earn nearly three times as much as production workers.

This wage gap arises even though about two-thirds of managers are not top executives but supervisors, team leaders, or intermediary executives (see Table A.2). We measure the number of workers a manager has under his charge (the span of control) with the ratio of non-managers to managers in each establishment. In half of the establishments, managers have a span of control lower than eight workers, and only one-fourth of establishments have managers with a span of control greater than seventeen workers. These results highlight that

[^13]most establishments assign a small span of control to their managers.

To summarize, we find substantial wage dispersion between managers and production workers. The literature on income inequality mainly attributes wage differences between groups to productivity-enhancing forces such as skill-biased technologies (Katz and Murphy, 1992; Autor, 2014), task-biased technologies (David et al., 2006), and trade specialization (Chetverikov et al., 2016). In this paper, we explore an additional force behind the wage dispersion between these occupations: heterogeneity in market competition.

## B. 4 Measuring Market Payroll Concentration

Our baseline measure of market concentration is the HHI. Given the employment $n_{i j o}$ and wage $w_{i j o}$ level at firm $i$ in a local labor market $j$ for occupation $o$, we define the HHI in the market as:

$$
\begin{align*}
\mathrm{HHI}_{j o} & :=\sum_{i=1}^{M_{j o}} s_{i j o}^{2}=\frac{1}{M_{j o}}+\sum_{i=1}^{M_{j o}}\left(s_{i j o}-\frac{1}{M_{j o}}\right)^{2},  \tag{20}\\
s_{i j o} & :=\frac{w_{i j o} n_{i j o}}{\sum_{i=1}^{M_{j o}} w_{i j o} n_{i j o}} . \tag{21}
\end{align*}
$$

Here, $M_{j o}$ is the number of establishments in market $j$ that hire workers in occupation $o$, and $s_{i j o}$ stands for the payroll share of the firm $i$. The HHI equals the average payroll market share weighted by the payroll share itself. The index ranges from $\frac{1}{M}$ to 1 , where a low value reflects low concentration or many firms having similar payroll shares. Note that this index gives more weight to larger establishments, especially penalizing markets where a few firms have a large share of the market payroll. The rightmost equality of Equation (20) shows that the HHI has an economically meaningful decomposition into two concentration sources. The first element involves the number of establishments in each market. All else being constant, increasing the number of establishments lowers the average establishment size in the market. The second element entails the dispersion level of payroll shares across establishments relative to the case in which they hold identical shares. All else being constant, increasing the dispersion in payroll shares leads to greater payroll concentration.

## C Appendix: Derivations

## C. 1 Labor Supply

## Microfoundations of the Nested CES Labor Supply

Closely following Berger et al. (2022), we provide a microfoundation of the labor supply curves in Equation (7). In particular, we show that this specification arises from a model in which individuals have heterogeneous idiosyncratic preferences for firms.

Consider a unit measure of ex-ante identical individuals indexed by $l \in[0,1]$ and a finite set of $J$ local labor markets each populated by $M_{j}$ firms. Suppose that workers derive utility according to the following specification:

$$
U_{l i j}=\log w_{i j}-\log y_{l i j}+\log B_{j}+\zeta_{l i j},
$$

where $\zeta_{l i j}$ is the idiosyncratic amenity that agent $l$ derives from working at firm $i j, w_{i j}$ stands for the wage that the agent earns by working $h_{l i j}$ hours at firm $i j$, the parameter $B_{j}$ represents an amenity from working in market $j$, and $y_{l i j}=w_{i j} h_{l i j}$ represents earnings. Given a parameter $\theta>0$, the following utility function represents the same preferences:

$$
\tilde{U}_{l i j}=(1+\theta) U_{l i j}=(1+\theta)\left(\log w_{i j}-\log y_{l i j}+\log B_{j}\right)+\tilde{\zeta}_{l i j},
$$

where the idiosyncratic amenity $\tilde{\zeta}_{l i j}$ is distributed according to a multi-variate Gumbel distribution:

$$
\begin{aligned}
F\left(\tilde{\zeta}_{i 1}, \ldots, \tilde{\zeta}_{i J}\right) & =\exp \left[-\sum_{j=1}^{J}\left(\sum_{i=1}^{M_{j}} e^{-\frac{\tilde{\zeta}_{i j}}{\rho}}\right)^{\rho}\right]=\exp \left[-\sum_{j=1}^{J}\left(\sum_{i=1}^{M_{j}} e^{-(1+\eta) \zeta_{i j}}\right)^{\frac{1+\theta}{1+\eta}}\right] \\
\rho & =\frac{1+\theta}{1+\eta}
\end{aligned}
$$

The parameter $\rho$ is a function of the correlation between the idiosyncratic amenities within each market $j$. Since the joint distribution of idiosyncratic amenities is a Generalized Extreme Value (GEV) distribution, then the probability that a worker $l$ chooses firm $i j$ has a
closed-form solution equal to: ${ }^{16}$

$$
\pi_{i j}=\underbrace{\frac{w_{i j}^{1+\eta}}{\sum_{i \in j} w_{i j}^{1+\eta}}}_{\text {Prob. worker chooses } i \mid j} \cdot \underbrace{\sum_{j=1}^{J}\left[\sum_{i \in j}\left(B_{j} w_{i j}\right)^{1+\eta}\right]^{\frac{1+\theta}{1+\eta}}}_{\text {Prob. worker chooses } j}
$$

Using the previous equation, we derive the firm-level labor supply:

$$
\begin{aligned}
n_{i j} & =\int_{0}^{1} \pi_{i j} h_{l i j} d F\left(y_{l}\right)=\frac{w_{i j}^{\eta}}{\sum_{i \in j} w_{i j}^{1+\eta}} \cdot \frac{\left[\sum_{i \in j}\left(B_{j} w_{i j}\right)^{1+\eta}\right]^{\frac{1+\theta}{1+\eta}}}{\sum_{j=1}^{J}\left[\sum_{i \in j}\left(B_{j} w_{i j}\right)^{1+\eta}\right]^{\frac{1+\theta}{1+\eta}}} \cdot \underbrace{\int_{0}^{1} w_{i j} h_{l i j} d F\left(y_{l}\right)}_{=Y}, \\
\Rightarrow n_{i j} & =\frac{w_{i j}^{\eta}}{\sum_{i \in j} w_{i j}^{1+\eta}} \cdot \frac{\left[\sum_{i \in j}\left(B_{j} w_{i j}\right)^{1+\eta}\right]^{\frac{1+\theta}{1+\eta}}}{\sum_{j=1}^{J}\left[\sum_{i \in j}\left(B_{j} w_{i j}\right)^{1+\eta}\right]^{\frac{1+\theta}{1+\eta}}} \cdot Y .
\end{aligned}
$$

To derive the expression of Equation (7), we first define the following wage and employment indexes:

$$
\begin{array}{ll}
\mathbf{w}_{j}=\left[\sum_{i \in j} w_{i j}^{1+\eta}\right]^{\frac{1}{1+\eta}}, & \mathbf{n}_{j}=\left[\sum_{i \in j} n_{i j}^{\frac{1+\eta}{\eta}}\right]^{\frac{\eta}{1+\eta}}, \\
\mathbf{W}=\left[\sum_{j=1}^{J}\left(B_{j} \mathbf{w}_{j}\right)^{1+\theta}\right]^{\frac{1}{1+\theta}}, & \mathbf{N}=\left[\sum_{j=1}^{J}\left(\frac{\mathbf{n}_{j}}{B_{j}}\right)^{\frac{1+\theta}{\theta}}\right]^{\frac{\theta}{1+\theta}} .
\end{array}
$$

[^14]Using these definitions and the previous labor supply curve, we show that $\mathbf{w}_{j} \mathbf{n}_{j}=\sum_{i \in j} w_{i j} n_{i j}$ :

$$
\begin{aligned}
\sum_{i \in j} w_{i j} n_{i j} & =\frac{\left[\sum_{i \in j}\left(B_{j} w_{i j}\right)^{1+\eta}\right]^{\frac{1+\theta}{1+\eta}}}{\sum_{j=1}^{J}\left[\sum_{i \in j}\left(B_{j} w_{i j}\right)^{1+\eta}\right]^{\frac{1+\theta}{1+\eta}} \cdot Y,} \\
& =\underbrace{\left[\sum_{i \in j} w_{i j}^{1+\eta}\right]^{\frac{1}{1+\eta}} \cdot \frac{\left[\sum_{i \in j} w_{i j}^{1+\eta}\right]^{\frac{\eta}{1+\eta}}}{\sum_{i \in j} w_{i j}^{1+\eta}} \cdot \frac{\left[\sum_{i \in j}\left(B_{j} w_{i j}\right)^{1+\eta}\right]^{\frac{1+\theta}{1+\eta}}}{\sum_{j=1}^{J}\left[\sum_{i \in j}\left(B_{j} w_{i j}\right)^{1+\eta}\right]^{\frac{1+\theta}{1+\eta}}} \cdot Y,}_{=1} \\
& =\left[\sum_{i \in j} w_{i j}^{1+\eta}\right]^{\frac{1}{1+\eta}} \cdot\left[\frac{\sum_{i \in j} w_{i j}^{1+\eta}}{\left[\sum_{i \in j} w_{i j}^{1+\eta}\right]^{\frac{1+\eta}{\eta}}} \cdot \frac{\left[\sum_{i \in j}\left(B_{j} w_{i j}\right)^{1+\eta}\right]^{\frac{1+\theta}{\eta}}}{\left(\sum_{j=1}^{J}\left[\sum_{i \in j}\left(B_{j} w_{i j}\right)^{1+\eta}\right]^{\frac{1+\theta}{1+\eta}}\right)^{\frac{1+\eta}{\eta}}} \cdot Y^{\frac{1+\eta}{\eta}}\right]^{\frac{\eta}{1+\eta}}, \\
& =\left[\sum_{i \in j} w_{i j}^{1+\eta}\right]^{\frac{1}{1+\eta}} \cdot\left[\sum_{i \in j}\left(\frac{w_{i j}^{\eta}}{\sum_{i \in j} w_{i j}^{1+\eta}} \cdot \frac{\left[\sum_{i \in j}\left(B_{j} w_{i j}\right)^{1+\eta}\right]^{\frac{1+\theta}{1+\eta}}}{\sum_{j=1}^{J}\left[\sum_{i \in j}\left(B_{j} w_{i j}\right)^{1+\eta}\right]^{\frac{1+\theta}{1+\eta}}} \cdot Y\right)^{\frac{1+\eta}{\eta}}\right]^{\frac{\eta}{1+\eta}} \\
& =\mathbf{w}_{j} \mathbf{n}_{j} .
\end{aligned}
$$

Next, we define $\tilde{w}_{i j}:=B_{j} w_{i j}$ and show that $Y:=\sum_{j} \mathbf{w}_{j} \mathbf{n}_{j}=\mathbf{W N}$

$$
\begin{aligned}
\mathbf{W N} & =\left[\sum_{j=1}^{J}\left(B_{j} \mathbf{w}_{j}\right)^{1+\theta}\right]^{\frac{1}{1+\theta}} \cdot\left[\sum_{j=1}^{J}\left(\frac{\mathbf{n}_{j}}{B_{j}}\right)^{\frac{1+\theta}{\theta}}\right]^{\frac{\theta}{1+\theta}}, \\
& =[\sum_{j=1}^{J} B_{j}^{1+\theta}(\underbrace{\left(\sum_{i \in j} w_{i j}^{1+\eta}\right)^{\frac{1+\theta}{1+\eta}}}_{=\mathbf{w}_{j}^{1+\theta}}]^{\frac{1}{1+\theta}} \cdot[\sum_{j=1}^{J}(\frac{1}{B_{j}} \cdot \frac{1}{\left[\sum_{i \in j} w_{i j}^{1+\eta}\right]^{\frac{1}{1+\eta}}} \cdot \underbrace{\sum_{i \in j}}_{\sum_{j=1}^{J}\left[\sum_{i \in j} \tilde{w}_{i j}^{1+\eta}\right]^{\frac{1+\theta}{1+\eta}}})^{\left.\frac{\left[\tilde{w}_{j}\right.}{1+\eta}\right]^{\frac{1+\theta}{1+\eta}}}, \\
& =\frac{\left[\sum_{j=1}^{J}\left(\sum_{i \in j} \tilde{w}_{i j}^{1+\eta}\right)^{\frac{1+\theta}{1+\eta}}\right]^{\frac{1}{1+\theta}} \cdot\left[\sum_{j=1}^{J}\left(\sum_{i \in j} \tilde{w}_{i j}^{1+\eta}\right)^{\frac{1+\theta}{1+\theta}}\right]^{\frac{\theta}{1+\theta}}}{\sum_{j=1}^{J}\left(\sum_{i \in j} \tilde{w}_{i j}^{1+\eta}\right)^{\frac{1+\theta}{1+\eta}}} \cdot Y, \\
& =Y:=\sum_{j} \mathbf{w}_{\mathbf{j}} \mathbf{n}_{\mathbf{j}} .
\end{aligned}
$$

Therefore, plugging the aforementioned two expressions into the firms' labor supply equation
yields the final expression in Equation (7):

$$
\begin{aligned}
n_{i j} & =\frac{w_{i j}^{\eta}}{\sum_{i \in j} w_{i j}^{1+\eta}} \cdot \frac{\left[\sum_{i \in j}\left(B_{j} w_{i j}\right)^{1+\eta}\right]^{\frac{1+\theta}{1+\eta}}}{\sum_{j=1}^{J}\left[\sum_{i \in j}\left(B_{j} w_{i j}\right)^{1+\eta}\right]^{\frac{1+\theta}{1+\eta}}} \cdot \mathbf{W N} \\
& =\frac{w_{i j}^{\eta}}{\mathbf{w}_{j}^{1+\eta}} \cdot \frac{B_{j}^{1+\theta} \mathbf{w}_{j}^{1+\theta}}{\sum_{j=1}^{J} B_{j}^{1+\theta} \mathbf{w}_{j}^{1+\theta}} \cdot \mathbf{W N} \\
& =\left(\frac{w_{i j}}{\mathbf{w}_{j}}\right)^{\eta} \cdot \frac{B_{j}^{1+\theta} \mathbf{w}_{j}^{\theta}}{\mathbf{W}^{1+\theta}} \cdot \mathbf{W N} \\
\Rightarrow n_{i j} & =B_{j}^{1+\theta} \cdot\left(\frac{w_{i j}}{\mathbf{w}_{\mathbf{j}}}\right)^{\eta} \cdot\left(\frac{\mathbf{w}_{j}}{\mathbf{W}}\right)^{\theta} \cdot \mathbf{N} .
\end{aligned}
$$

To get the expression for the inverse labor supply curve, we first need to compute the marketlevel supply curve:

$$
\begin{aligned}
\mathbf{n}_{j} & =\left[\sum_{i \in j} n_{i j}^{\frac{1+\eta}{\eta}}\right]^{\frac{\eta}{1+\eta}} \\
& =\left[\sum_{i \in j}\left(\left(\frac{w_{i j}}{\mathbf{w}_{\mathbf{j}}}\right)^{\eta} \cdot\left(\frac{\mathbf{w}_{j}}{\mathbf{W}}\right)^{\theta} \cdot B_{j}^{1+\theta} \cdot \mathbf{N}\right)^{\frac{1+\eta}{\eta}}\right]^{\frac{\eta}{1+\eta}}, \\
& =\left[\sum_{i \in j} w_{i j}^{1+\eta}\right]^{\frac{\eta}{1+\eta}} \cdot \frac{\mathbf{w}_{j}^{\theta}}{\mathbf{w}_{j}^{\eta} \mathbf{W}^{\theta}} \cdot \mathbf{N} B_{j}^{1+\theta}, \\
\Rightarrow \mathbf{n}_{j} & =\left(\frac{\mathbf{w}_{j}}{\mathbf{W}}\right)^{\theta} \cdot \mathbf{N} B_{j}^{1+\theta}
\end{aligned}
$$

Then, we rearrange the market-level and the firms' labor supply curves:

$$
\begin{aligned}
& \mathbf{w}_{j}=\left(\frac{\mathbf{n}_{j}}{\mathbf{N}}\right)^{\frac{1}{\theta}} \cdot \frac{\mathbf{W}}{B_{j}^{\frac{1+\theta}{\theta}}}, \\
& w_{i j}=\left(\frac{n_{i j}}{\mathbf{n}_{j}}\right)^{\frac{1}{\eta}} \cdot \mathbf{w}_{j} .
\end{aligned}
$$

Lastly, plugging the inverse market-level supply curve into the inverse labor supply curve yields the final expression in Equation (7):

$$
w_{i j}=\frac{1}{B_{j}^{\frac{1+\theta}{\theta}}} \cdot\left(\frac{n_{i j}}{\mathbf{n}_{j}}\right)^{\frac{1}{\eta}} \cdot\left(\frac{\mathbf{n}_{j}}{\mathbf{N}}\right)^{\frac{1}{\theta}} \cdot \mathbf{W}
$$

## Solving the Household Problem

This section solves the household problem of Section 3. Each household type $o \in\{w, m\}$ choose consumption $\left\{c_{i j o}\right\}$ and the amount of labor supply $\left\{n_{i j o}\right\}$ to each firm to maximize
their utility, taking as given wages:

$$
\mathcal{U}_{o}=\max _{n_{i j o}, c_{i j o}} \quad \mathbf{C}_{o}-\phi_{o} \frac{\mathbf{N}_{o}^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}}
$$

subject to the household's budget constraint: ${ }^{17}$

$$
\mathbf{C}_{o}=\int_{0}^{1} \sum_{i=1}^{M_{j}} w_{i j o} n_{i j o} d j
$$

where we define the aggregate consumption and labor supply indexes as

$$
\begin{aligned}
& \mathbf{C}_{o}:=\int_{0}^{1} \sum_{i=1}^{M_{j}} c_{i j o} d j \\
& \mathbf{N}_{\mathbf{o}}:=\left[\int_{0}^{1}\left(\frac{\mathbf{n}_{\mathbf{j o}}}{B_{j o}}\right)^{\frac{\theta_{o}+1}{\theta_{o}}} d j\right]^{\frac{\theta_{o}}{\theta_{o}+1}} \mathbf{n}_{\mathbf{j o}}:=\left[\sum_{i=1}^{M_{j}} n_{i j o}^{\frac{\eta_{o}+1}{\eta_{o}}}\right]^{\frac{\eta_{o}}{\eta_{o}+1}}, \quad \eta_{o}>\theta_{o}>0 .
\end{aligned}
$$

The Lagrangian of this maximization problem is:

$$
\mathcal{L}\left(\left\{c_{i j o}\right\},\left\{n_{i j o}\right\}, \lambda\right)=U_{o}\left(\left\{c_{i j o}\right\},\left\{n_{i j o}\right\}\right)+\lambda\left(\int_{0}^{1} \sum_{i=1}^{M_{j}} w_{i j o} n_{i j o} d j-\mathbf{C}_{o}\right) .
$$

The first-order conditions associated with this problem are:

$$
\begin{align*}
& \frac{\partial \mathcal{L}}{\partial c_{i j o}}=0 \Longleftrightarrow \lambda=\frac{\partial U_{o}}{\partial \mathbf{C}_{o}} \cdot \frac{\partial \mathbf{C}_{o}}{\partial c_{i j o}} \Longleftrightarrow \lambda=1 \quad \forall i j o,  \tag{C.1}\\
& \frac{\partial \mathcal{L}}{\partial n_{i j o}}=0 \quad \Longleftrightarrow \lambda w_{i j o}=\frac{\partial U_{o}}{\partial \mathbf{N}_{o}} \cdot \frac{\partial \mathbf{N}_{o}}{\partial \mathbf{n}_{j o}} \cdot \frac{\partial \mathbf{n}_{j o}}{\partial n_{i j o}} \quad \forall i j o . \tag{C.2}
\end{align*}
$$

Substituting Equation (C.1) into Equation (C.2) implies:

$$
\begin{equation*}
w_{i j o}=-\frac{\partial U_{o}}{\partial \mathbf{N}_{o}} \cdot \frac{\partial \mathbf{N}_{o}}{\partial \mathbf{n}_{j o}} \cdot \frac{\partial \mathbf{n}_{j o}}{\partial n_{i j o}} \tag{C.3}
\end{equation*}
$$

where each component of the last equation is equal to:

$$
\begin{align*}
-\frac{\partial U_{o}}{\partial \mathbf{N}_{o}} & =\phi_{o} \mathbf{N}^{\frac{1}{\gamma}}  \tag{C.4}\\
\frac{\partial \mathbf{N}_{o}}{\partial \mathbf{n}_{j o}} & =\left(\frac{\mathbf{n}_{j o}}{\mathbf{N}_{o}}\right)^{\frac{1}{\theta}} \cdot \frac{1}{B_{j}^{\frac{1+\theta}{\theta}}}  \tag{C.5}\\
\frac{\partial \mathbf{n}_{j o}}{\partial n_{i j o}} & =\left(\frac{n_{i j o}}{\mathbf{n}_{j o}}\right)^{\frac{1}{\eta}} \tag{C.6}
\end{align*}
$$

[^15]Therefore, plugging the previous expressions into Equation (C.3):

$$
\begin{equation*}
w_{i j o}=\left(\frac{n_{i j o}}{\mathbf{n}_{j o}}\right)^{\frac{1}{\eta}} \cdot\left(\frac{\mathbf{n}_{j o}}{\mathbf{N}_{o}}\right)^{\frac{1}{\theta}} \cdot \frac{1}{B_{j}^{\frac{1+\theta}{\theta}}} \cdot\left(-\frac{\partial U_{o}}{\partial \mathbf{N}_{o}}\right) \tag{C.7}
\end{equation*}
$$

To get the final expression of the firms' labor supply curve, we need to show that under optimality the aggregate wage is equal to the marginal disutility of aggregate labor supply. First, we define the market and aggregate wage indexes as follows:

$$
\begin{align*}
& \mathbf{w}_{j o}=\left[\sum_{i \in j} w_{i j}^{\eta}\right]^{\frac{1}{1+\eta}}  \tag{C.8}\\
& \mathbf{W}_{o}=\left[\sum_{j=1}^{J}\left(B_{j} \mathbf{w}_{j}\right)^{1+\theta}\right]^{\frac{1}{1+\theta}} . \tag{C.9}
\end{align*}
$$

Substituting Equation (C.7) into Equation (C.8) implies:

$$
\begin{equation*}
\mathbf{w}_{j o}=\left(\frac{\mathbf{n}_{j o}}{\mathbf{N}_{o}}\right)^{\frac{1}{\theta}} \cdot \frac{1}{B_{j}^{\frac{1+\theta}{\theta}}} \cdot\left(-\frac{\partial U_{o}}{\partial \mathbf{N}_{o}}\right) . \tag{C.10}
\end{equation*}
$$

Then, substituting the last equation into Equation (C.9) yields the desired result:

$$
\begin{equation*}
\mathbf{W}_{o}=-\frac{\partial U_{o}}{\partial \mathbf{N}_{o}} \tag{C.11}
\end{equation*}
$$

Moreover, we derive the expression for the aggregate labor supply disutility in Equation (6) by plugging Equation (C.4) into Equation (C.11) and rearranging:

$$
\mathbf{N}_{o}=\left(\frac{\mathbf{W}_{o}}{\phi_{o}}\right)^{\gamma} .
$$

Finally, we get the final expression for the firms' labor supply curve. Note that Equation (C.7) and Equation (C.11) imply the inverse firms' labor supply curve in Equation (7):

$$
w_{i j o}=\frac{1}{B_{j}^{\frac{1+\theta}{\theta}}} \cdot\left(\frac{n_{i j o}}{\mathbf{n}_{j o}}\right)^{\frac{1}{\eta}} \cdot\left(\frac{\mathbf{n}_{j o}}{\mathbf{N}_{o}}\right)^{\frac{1}{\theta}} \cdot \mathbf{W}_{o}
$$

Moreover, substituting Equation (C.11) into Equation (C.10) and rearranging imply:

$$
\mathbf{n}_{j}=\left(\frac{\mathbf{w}_{j}}{\mathbf{W}}\right)^{\theta} \cdot \mathbf{N} B_{j}^{1+\theta}
$$

and substituting Equation (C.10) into Equation (C.7) and rearranging imply:

$$
n_{i j o}=\left(\frac{w_{i j o}}{\mathbf{w}_{j}}\right)^{\eta} \cdot \mathbf{n}_{j} .
$$

Hence, using the last two equations yields the expression for the firms' labor supply curve in Equation (7):

$$
n_{i j}=B_{j}^{1+\theta} \cdot\left(\frac{w_{i j}}{\mathbf{w}_{\mathbf{j}}}\right)^{\eta} \cdot\left(\frac{\mathbf{w}_{j}}{\mathbf{W}}\right)^{\theta} \cdot \mathbf{N} .
$$

## C. 2 Labor Demand

This section solves the firm problem of Section (3) for single-layer firms. Since we solve the organizational problem of choosing the number of layers numerically, we proceed to analytically solve the profit maximization problem subsequent to adopting the organizational structure. Moreover, we omit the problem for multi-layer firms for illustrative purposes, as the additional complexity added by this problem compared to the problem of single-layer firms only involves an expanded set of scenarios to analyze.

When firms adopt a single-layer structure, they also choose the measure of production workers $n_{i j w}$ to maximize profits, given the employment policies of their local competitors, $n_{-i j w}^{*}$. In particular, they solve:

$$
\pi(z, 1)=\max _{n_{i j w}} y(z, 1)-w_{i j w}\left(n_{i j w}, n_{-i j w}^{*}, \mathbf{N}_{w}, \mathbf{W}_{w}\right) n_{i j w}
$$

subject to the inverse labor supply curve of production workers and minimum wages:

$$
\begin{aligned}
& w_{i j w}\left(n_{i j w}, n_{-i j w}^{*}, \mathbf{N}_{w}, \mathbf{W}_{w}\right)=\left(\frac{1}{B_{j w}}\right)^{\frac{1+\theta_{o}}{\theta_{o}}}\left(\frac{n_{i j w}}{\mathbf{n}_{j w}\left(n_{i j w}, n_{-i j w}^{*}\right)}\right)^{\frac{1}{\eta_{w}}}\left(\frac{\mathbf{n}_{j w}\left(n_{i j w}, n_{-i j w}^{*}\right)}{\mathbf{N}_{w}}\right)^{\frac{1}{\theta_{w}}} \mathbf{W}_{w} \\
& \mathbf{n}_{j w}\left(n_{i j w}, n_{-i j w}^{*}\right)=\left[n_{i j w}^{\frac{1+\eta_{w}}{\eta_{w}}}+\sum_{k \neq i} n_{k j w}^{*} \frac{1+\eta_{w}}{\eta_{w}}\right]^{\frac{\eta_{w}}{1+\eta_{w}}} \\
& w_{i j w} \geq \underline{w}
\end{aligned}
$$

The associated Lagrangian function is:

$$
\mathcal{L}\left(n_{i j w}, \mu\right)=y(z, 1)-w_{i j w} n_{i j w}+\nu \cdot\left(w_{i j w}-\underline{w}\right) .
$$

To ease notation, we omit that the following conditions hold when employment is optimal and that the firm internalizes an inverse labor supply that is a function of the labor supply
of all competitors within the market. The system of Kuhn-Tucker conditions is given by:

$$
\begin{align*}
\frac{\partial \mathcal{L}}{\partial n_{i j w}} & =0 \Longleftrightarrow \frac{\partial y(z, 1)}{\partial n_{i j w}}+\nu=\frac{\partial w_{i j w}}{\partial n_{i j w}} \cdot n_{i j w}+w_{i j w}  \tag{C.12}\\
\nu \cdot\left(w_{i j w}-\underline{w}\right) & =0,  \tag{C.13}\\
\nu & \geq 0,  \tag{C.14}\\
w_{i j w} & =\max (\underline{w}, \underbrace{\left.\left(\frac{1}{B_{j w}}\right)^{\frac{1+\theta_{o}}{\theta_{o}}}\left(\frac{n_{i j w}}{\mathbf{n}_{j w}}\right)^{\frac{1}{\eta_{w}}}\left(\frac{\mathbf{n}_{j w}}{\mathbf{N}_{\mathbf{w}}}\right)^{\frac{1}{\theta_{w}}} \mathbf{W}_{w}\right)}_{=\tilde{w}_{i j w}, \text { i.e., unconstrained labor supply curve }} \text { ), }  \tag{C.15}\\
\mathbf{n}_{j w} & =\left[n_{n_{i j w}^{\frac{1+\eta_{w}}{\eta_{w}}}}+\sum_{k \neq i} n_{k j w}^{*} \frac{1+\eta_{w}}{\eta_{w}}\right]^{\frac{\eta_{w}}{1+\eta_{w}}} . \tag{C.16}
\end{align*}
$$

To solve the maximization problem, we break the problem into three different cases.
Case I: The minimum wage is not binding. Suppose the case when the minimum wage is not binding $w_{i j w}^{*}>\underline{w}$. Then, Equation (C.13) implies that $\nu=0$, and Equation (C.12) is given by:

$$
\begin{aligned}
\left.\frac{\partial y(z, 1)}{\partial n_{i j w}}\right|_{n_{i j w}^{*}} & =\left.n_{i j w}^{*} \cdot \frac{\partial w_{i j w}}{\partial n_{i j w}}\right|_{n_{i j w}^{*}}+w_{i j w}^{*}, \\
\left.\frac{\partial y(z, 1)}{\partial n_{i j w}}\right|_{n_{i j w}^{*}} & =\frac{w_{i j w}^{*}}{\varepsilon_{i j w}^{*}}+w_{i j w}^{*} \\
\Rightarrow w_{i j w}^{*} & =\left.\mu_{i j w}^{*} \cdot \frac{\partial y(z, 1)}{\partial n_{i j w}}\right|_{n_{i j w}^{*}}
\end{aligned}
$$

where $\varepsilon_{i j w}$ is the structural elasticity of labor supply and $\mu_{i j w}$ represents the wage markdown:

$$
\begin{aligned}
\varepsilon_{i j o} & =\left[\frac{\partial \log w_{i j o}}{\partial \log n_{i j o}}\right]^{-1} \\
\mu_{i j w} & =\frac{\varepsilon_{i j w}}{\varepsilon_{i j w}+1} \in[0,1] .
\end{aligned}
$$

In Appendix C. 3 we show that the structural elasticity of labor supply has a closed-form solution given by:

$$
\begin{equation*}
\varepsilon_{i j o}\left(s_{i j o}\right)=\left[\frac{1}{\eta_{o}}+\left(\frac{1}{\theta_{o}}-\frac{1}{\eta_{o}}\right) \frac{\partial \log \mathbf{n}_{j o}}{\partial \log n_{i j o}}\right]^{-1}=\left[\frac{1}{\eta_{o}}+\left(\frac{1}{\theta_{o}}-\frac{1}{\eta_{o}}\right) s_{i j o}\right]^{-1} \tag{C.17}
\end{equation*}
$$

where $s_{i j o}$ is the payroll share of firm $i$ in market $j$ :

$$
\begin{equation*}
s_{i j o}:=\frac{w_{i j o} n_{i j o}}{\sum_{i \in j} w_{i j o} n_{i j o}} \tag{C.18}
\end{equation*}
$$

Case II: The minimum wage is binding, and labor supply equals labor demand. Suppose the case when the minimum wage is binding $w_{i j w}^{*}=\underline{w}$ and labor supply equals labor demand, that is, Equation (C.15) satisfies:

$$
\begin{equation*}
w_{i j w}^{*}=\underline{w}=\left(\frac{1}{B_{j w}}\right)^{\frac{1+\theta_{o}}{\theta_{o}}}\left(\frac{n_{i j w}^{*}}{\mathbf{n}_{j w}^{*}}\right)^{\frac{1}{\eta_{w}}}\left(\frac{\mathbf{n}_{j w}^{*}}{\mathbf{N}_{\mathbf{w}}}\right)^{\frac{1}{\theta_{w}}} \mathbf{W}_{w} \tag{C.19}
\end{equation*}
$$

Then, the optimal level of employment is given by Equation (C.19). Moreover, the Lagrange multiplier associated with the inequality constraint may not be binding, i.e., $\nu \geq 0$. Thus, Equation (C.12) implies that the marginal revenue must be smaller or equal than the marginal cost. In contrast, the marginal revenue must be greater or equal than the minimum wage. We prove this by contradiction. Suppose $\underline{w}>\operatorname{mrpl}\left(n_{i j w}^{*}\right)$. Since the unconstrained labor supply curve is strictly increasing in labor, then Equation (C.15) implies that $w\left(n_{i j w}\right)=\underline{w} \quad \forall n_{i j w}<n_{i j w}^{*}$. Thus, the marginal cost is also equal to the minimum wage within this employment range: $\operatorname{mc}\left(n_{i j w}\right)=\underline{w} \forall n_{i j w}<n_{i j w}^{*}$. Moreover, since the marginal revenue of labor is strictly decreasing in labor units, then there exists a threshold $n_{i j w}^{\prime}<n_{i j w}^{*}$ for which $\operatorname{mrpl}\left(n_{i j w}^{\prime}\right)=\underline{w}$ and $\operatorname{mrpl}\left(n_{i j w}\right)<\underline{w} \quad \forall n_{i j w} \in\left(n_{i j w}^{\prime}, n_{i j w}^{*}\right]$. However, this implies that $n_{i j w}^{\prime}$ is feasible and more profitable than $n_{i j w}^{*}$ because any unit between them yields negative profits, i.e., their marginal cost is higher than their marginal revenue, which contradicts that $n_{i j w}^{*}$ is optimal. Hence, it must be the case that $\underline{w} \leq \operatorname{mrpl}\left(n_{i j w}^{*}\right)$.

Overall, the previous results imply that:

$$
\begin{align*}
& w_{i j w}^{*}=\underline{w}  \tag{C.20}\\
&\left.\frac{\partial y(z, 1)}{\partial n_{i j w}}\right|_{n_{i j w}^{*}} \leq\left.\frac{\partial w_{i j w}}{\partial n_{i j w}}\right|_{n_{i j w}^{*}}+\underline{w},  \tag{C.21}\\
&\left.\frac{\partial y(z, 1)}{\partial n_{i j w}}\right|_{n_{i j w}^{*}} \geq \underline{w} . \tag{C.22}
\end{align*}
$$

Here, the markdown does not have a closed-form solution but is given by:

$$
\begin{equation*}
\mu_{i j w}=\frac{\underline{w}}{\left.\frac{\partial y(z, 1)}{\partial n_{i j w}}\right|_{n_{i j w}^{*}}} \in[0,1] . \tag{C.23}
\end{equation*}
$$

Case III: The minimum wage is binding, and labor supply exceeds labor demand. Suppose the case when the minimum wage is binding $w_{i j w}^{*}=\underline{w}$ and labor supply excess labor demand,
that is, Equation (C.15) satisfies:

$$
\begin{equation*}
w_{i j w}^{*}=\underline{w}>\left(\frac{1}{B_{j w}}\right)^{\frac{1+\theta_{o}}{\theta_{o}}}\left(\frac{n_{i j w}^{*}}{\mathbf{n}_{j w}^{*}}\right)^{\frac{1}{\eta_{w}}}\left(\frac{\mathbf{n}_{j w}^{*}}{\mathbf{N}_{\mathbf{w}}}\right)^{\frac{1}{\theta_{w}}} \mathbf{W}_{w} . \tag{C.24}
\end{equation*}
$$

Next, we note that the previous fact implies that the marginal cost of an additional hire is the minimum wage. Graphically, the wage function is flat in a neighborhood of the optimal labor choice $n_{i j w}^{*}$, as displayed in Figure A.3. To prove it mathematically, we also rely on the fact that the unconstrained labor supply curve is strictly increasing in labor. Thus, $\tilde{w}_{i j w}\left(\tilde{n}_{i j w}\right)=\underline{w}$ for $\tilde{n}_{i j w}>n_{i j w}^{*}$. Then, there exists $\bar{\varepsilon}>0$ such that $\varepsilon \in[0, \bar{\varepsilon}]$ and $n_{i j w}^{\prime}=n_{i j w}^{*}+\varepsilon<\tilde{n}_{i j w}$ for which $w_{i j w}^{*}\left(n_{i j w}^{\prime}\right)=\underline{w}$. As a result, $\frac{\partial w_{i j w}}{\partial n_{i j w}}=0$.

Moreover, this case also implies that $\nu=0$. We prove this by contradiction. Suppose $\nu>0$, then Equation (C.12) implies that $\left.\frac{\partial y(z, 1)}{\partial n_{i j w}}\right|_{n_{i j w}^{*}}<\underline{w}$. Since we assume that the marginal productivity is strictly decreasing in labor, there exists $\varepsilon>0$ such that $n_{i j w}^{\prime \prime}=n_{i j w}^{*}-\varepsilon$ and $\left.\frac{\partial y(z, 1)}{\partial n_{i j w}}\right|_{n_{i j w}^{\prime \prime}}=\underline{w}>\left.\frac{\partial y(z, 1)}{\partial n_{i j w}}\right|_{n_{i j w}^{*}}$. Then, choosing $n_{i j w}^{\prime \prime}$ and paying them $\underline{w}$ is feasible and more profitable because it raises revenue while keeping costs fixed. This is a contradiction because $n_{i j w}^{*}$ is optimal. Hence, it must be $\nu=0$.

Therefore, the aforementioned two results and Equation (C.12) imply that labor demand satisfies:

$$
\begin{equation*}
w_{i j w}^{*}=\underline{w}=\left.\frac{\partial y(z, 1)}{\partial n_{i j w}}\right|_{n_{i j w}^{*}}, \quad \mu_{i j w}=1 . \tag{C.25}
\end{equation*}
$$

Where the markdown equals one by definition.

## C. 3 Structural Elasticity of Labor Supply

To show that the structural elasticity has a closed-form solution, consider the log transformation of the inverse labor supply curve in Equation (7):
$\log \left(w\left(n_{i j o}, n_{-i j o}^{*}, \mathbf{W}_{o}, \mathbf{N}_{o}\right)\right)=\frac{1}{\eta_{o}} \log \left(n_{i j o}\right)+\left(\frac{1}{\theta_{o}}-\frac{1}{\eta_{o}}\right) \log \left(\mathbf{n}_{j o}\right)-\frac{1}{\theta_{o}} \log \left(\mathbf{N}_{o}\right)+\log \left(\mathbf{W}_{o}\right)-\frac{1+\theta_{o}}{\theta_{o}} \log \left(B_{j o}\right)$.
Thus,

$$
\frac{\partial \log \left(w\left(n_{i j o}, n_{-i j o}^{*}, \mathbf{W}_{o}, \mathbf{N}_{o}\right)\right)}{\partial \log \left(n_{i j o}\right)}=\frac{1}{\eta_{o}}-\left(\frac{1}{\theta_{o}}-\frac{1}{\eta_{o}}\right) \frac{\partial \log \left(\mathbf{n}_{j o}\right)}{\partial \log \left(n_{i j o}\right)}
$$

Note that the derivative of the economy-wide variables with respect to the firm's employment is zero because firms are atomistic with respect to the economy. Moreover, the definition of the market labor supply disutility index implies that:

$$
\begin{aligned}
\frac{\partial \log \left(\mathbf{n}_{j o}\right)}{\partial \log \left(n_{i j o}\right)} & =\frac{\partial \mathbf{n}_{j o}}{\partial n_{i j o}} \cdot \frac{n_{i j o}}{\mathbf{n}_{j o}} \\
& =n_{i j w}^{\frac{1+\eta_{o}}{\eta_{o}}} \cdot \mathbf{n}_{j o}^{-1} \cdot\left(\sum_{i \in j} n_{i j o}^{\frac{\eta_{o}+1}{\eta_{o}}}\right)^{-\frac{1}{\eta_{o}+1}} \\
& =\left(\frac{n_{i j o}}{\mathbf{n}_{j o}}\right)^{\frac{1+\eta_{o}}{\eta_{o}}}
\end{aligned}
$$

Therefore,

$$
\frac{\partial \log \left(w\left(n_{i j o}, n_{-i j o}^{*}, \mathbf{W}_{o}, \mathbf{N}_{o}\right)\right)}{\partial \log \left(n_{i j o}\right)}=\frac{1}{\eta_{o}}-\left(\frac{1}{\theta_{o}}-\frac{1}{\eta_{o}}\right) \cdot\left(\frac{n_{i j o}}{\mathbf{n}_{j o}}\right)^{\frac{1+\eta_{o}}{\eta_{o}}}
$$

Next, we show that the last fraction is equal to the payroll share of the firm $i$ in market $j$. In particular, defining the payroll share and substituting the inverse labor supply curve:

$$
\begin{aligned}
s_{i j o} & =\frac{w_{i j o} n_{i j o}}{\sum_{i \in j} w_{i j o} n_{i j o}}=\frac{n_{i j o}^{\frac{1+\eta_{o}}{\eta_{o}}}}{\sum_{i \in j} n_{i j o}^{\frac{1+\eta_{o}}{\eta_{o}}}}, \\
& =\left(\frac{n_{i j o}}{\mathbf{n}_{j o}}\right)^{\frac{1+\eta_{o}}{\eta_{o}}}
\end{aligned}
$$

Hence, the structural labor supply elasticity is given by:

$$
\varepsilon_{i j o}:=\left[\frac{\partial \log w\left(n_{i j o}, n_{-i j o}^{*}, \mathbf{W}_{o}, \mathbf{N}_{o}\right)}{\partial \log n_{i j o}}\right]^{-1}=\left[\frac{1}{\eta_{o}}-\left(\frac{1}{\theta_{o}}-\frac{1}{\eta_{o}}\right) s_{i j o}\right]^{-1}
$$

We take the concept of "structural" from Berger et al. (2022). The motivation for this concept is twofold. First, it arises from a structural macroeconomic model with firm granularity and strategic interaction in labor demand. In these models, the structural elasticity is the welfare-relevant variable of the model because its distribution determines the distribution of wage markdowns. Second, it is useful to distinguish the concept from the more commonly estimated reduced-form labor supply elasticity. On the one hand, the structural elasticity is the labor supply elasticity faced by a firm that internalizes the employment responses of its competitors within the local labor market. It measures the percent change in the firm's
labor supply due to an increase of one percent in the firm's wages, holding its competitors' employment constant. On the other hand, the reduced-form elasticity measures the percent change in the firm's labor supply due to an increase of one percent in the firm's wages. Thus, in our model, this variable includes the effect of the response of the firm's competitors on the firm's own wage. For example, when a firm receives an idiosyncratic positive shock and increases labor demand, Cournot competition implies that the firm's competitors best respond by decreasing labor demand, which also leads the shocked firm to best respond and increase its quantity of labor demand and so on.

## D Appendix: Algorithm

The solution of the equilibrium consists of a fixed point in wages. We solve the problem for 20 different firm productivity grids and 1,000 markets. To ease the algorithm's interpretation, we first describe the equilibrium solution in the absence of minimum wages.

## D. 1 Algorithm with No Minimum Wages

The idea of the algorithm is that whenever a firm faces an excess of labor demand for one occupation, then we smoothly increase its wage. In contrast, we smoothly decrease it whenever it faces an excess of labor supply. Thus, the algorithm always converges as long as the labor supply curve firms face is strictly increasing and the marginal revenue is strictly decreasing in employment.

We initialize the algorithm by guessing a vector of wages $\left\{w_{i j o}^{(0)}\right\}_{\forall i j o}$. Consider iteration $k$ :

## 1. Compute labor supply.

Note that firms' wages $w_{i j o}^{(k)}$ are enough to get $\mathbf{w}_{j o}^{(k)}, \mathbf{W}_{o}^{(k)}$, and $\mathbf{N}_{o}^{(k)}$ from Equations (6) and (8). Then, we compute the labor supply to each firm $n_{i j o}^{s,(k)}$ by substituting the previous variables into Equation (7).

## 2. Compute organizational choice.

Market clearing implies that labor supply is equal to labor demand in equilibrium. Thus, labor supply is equal to labor demand at equilibrium wages $n_{i j o}^{d,(k)}=n_{i j o}^{s,(k)}$.

Then, we use wages $\left\{w_{i j o}^{(k)}\right\}_{\forall i j o}$ and labor demands $\left\{n_{i j o}^{d,(k)}\right\}_{\forall i j o}$ to compute the optimal organizational choice $\ell^{(k)}$ for all firms using the same method that we describe in the previous subsection. Note that wages and employment change when we update the firm's optimal organizational structure, e.g., we set the wage and employment of managers to zero for single-layer firms: $w_{i j m}^{(k)}=0$ and $n_{i j m}^{s,(k)}=n_{i j m}^{d,(k)}=0$ if $\ell^{(k)}=1$.

## 3. Compute markdowns.

We compute the payroll market share $s_{i j o}^{(k)}$ using wages $w_{i j o}^{(k)}$ and labor demand $n_{i j o}^{d,(k)}$ of all firms in market $j$. Then, we compute the firm's structural elasticity $\varepsilon_{i j o}^{(k)}$ and markdown $\mu_{i j o}^{(k)}$ from Equations (12)-(13).

## 4. Compute wages from labor demand FOCs.

For each occupation, we use the optimal organization $\ell^{(k)}$ and labor demand $n_{i j o}^{d,(k)}$ of each firm to compute its marginal revenue product of labor:

$$
\operatorname{mrpl}_{i j o}^{(k)}=\left.\frac{\partial y\left(z, \ell^{(k)}\right)}{\partial n_{i j o}}\right|_{n_{i j o}^{d,(k)}}
$$

Then, we update the occupation-specific wages of all firms as:

$$
w_{i j o}^{\prime(k)}= \begin{cases}\mu_{i j o}^{(k)} \cdot \operatorname{mrpl}_{i j o}^{(k)}, & \text { if } n_{i j o}^{d,(k)}>0 \\ 0, & \text { if } n_{i j o}^{d,(k)}=0\end{cases}
$$

## 5. Iteration.

Iterate over (1) to (4) until convergence of wages. Whenever max $\left\{\operatorname{abs}\left(w_{i j o}^{(k)}-w_{i j o}^{\prime(k)}\right)\right\}>$ tol, we update wages with the following criterion:

$$
w_{i j o}^{(k+1)}=\rho w_{i j o}^{(k)}+(1-\rho) w_{i j o}^{\prime(k)} \quad \text { for } \quad \rho \in(0,1) .
$$

## D. 2 Computing the Optimal Organization Structure

Calculating the maximum profits a firm could earn by opting for the off-equilibrium organizational structure is the main challenge when determining a firm's optimal organizational structure. This is because computing off-equilibrium profits in each iteration would be overly computationally expensive. To address this issue, we proceed as follows.

We start by setting a slightly higher tolerance level than the main algorithm's. When the distance between the initial and predicted wages exceeds this tolerance, we assign the following off-equilibrium wages and employment to each firm:

- If the firm began the iteration as a multi-layer organization, we use its equilibrium wage and employment levels for production workers to calculate its profits as a singlelayer organization.
- If the firm began the iteration as a single-layer organization, we use the equilibrium wage and employment levels for production workers and managers of its nearest competitor to compute the firm's profits as a multi-layer organization. When there exist multi-layer firms within the same market, the nearest competitor is the multi-layer firm in the market located in the closest productivity bin. If no multi-layer firms are within the same market, we assign economy-wide minimum wage and employment levels for both occupations. Finally, we utilize this information to compute the firms' profits onand off-equilibrium for each organizational structure and solve the Problem (9).

Next, when the distance between the initial and predicted wages falls below this tolerance, it indicates that the algorithm is close to converging. In such cases, we compute the actual wage and employment levels for each firm's off-equilibrium organizational structure using a numerical solution. Specifically, we calculate off-equilibrium profits by numerically solving Problem (10) for multi-layer firms and Problem (11) for single-layer firms. With this information and the optimal profits for firms in equilibrium, we determine the optimal organizational structure as defined in Problem (9). This step in the algorithm aims to correct any potential misassignments of optimal organizational structures to firms, as we rely on information from their competitors. Given that this step is computationally intensive, we perform it only once. Following a single implementation, we continue with the previous method that utilizes information from competitors until convergence. ${ }^{18}$

After computing the optimal organizational structure, we update wages within the same iteration $k$. If the firm is initially multi-layer and we find a deviation that makes being

[^16]single-layer more profitable, we set managerial wages and employment to zero $w_{i j m}^{(k)}=0$ and $n_{i j m}^{(k)}=0$. For production workers, we set their wages and employment to the deviation values $w_{i j w}^{(k)}=w^{\prime}$ and $n_{i j w}^{(k)}=n^{\prime}$. If the firm is initially single-layer and we find a deviation that makes being multi-layer more profitable, we set both wages and employment to their deviation values $w_{i j o}^{(k)}=w^{\prime}$ and $n_{i j o}^{(k)}=n^{\prime} \forall o \in\{w, m\}$. Note that the deviation may originate from competitors or the optimal choice, but it results in higher profitability in either case.

Overall, this numerical solution for the optimal organization structure of firms performs well. After convergence, we observe an optimal deviation from the equilibrium organizational choice for 0.34 percent of firms. This error arises because an optimal deviation implies that only one firm deviates from equilibrium while its local market competitors maintain constant labor demand. However, in practice, our algorithm solution often implies that more than one firm deviates, for instance, in markets with multiple firms. Therefore, implementing such an optimal deviation for those firms becomes impractical.

## D. 3 Algorithm with Minimum Wages

We solve the equilibrium with minimum wages using a shadow wage approach as in Berger et al. (2023b). This approach is useful to deal with non-market-clearing wages and assumes that workers at constrained firms perceive a lower wage whenever an excess of labor supply exists at the minimum wage. In particular, the shadow or perceived wage is the wage for which the firm's labor supply equals its labor demand at the minimum wage. This implies that the excess labor supply at the minimum wage is reallocated towards other firms.

We initialize the algorithm by guessing a vector of wages $\left\{w_{i j o}^{(0)}\right\}_{\forall i j o}$ such that the minimum wage is not binding for any of the firms in the first iteration. Thus, we set the initial vector of shadow wages $\left\{\tilde{w}_{i j o}^{(0)}\right\}_{\forall i j o}$ equal to the initial vector of wages. Consider iteration $k$ :

## 1. Compute labor supply.

Note that shadow wages are enough to get $\tilde{\mathbf{w}}_{j o}^{(k)}$, $\tilde{\mathbf{W}}_{o}^{(k)}$, and $\tilde{\mathbf{N}}_{o}^{(k)}$ from Equations (6) and (8). Then, we compute the labor supply to each firm $n_{i j o}^{s,(k)}$ by substituting the previous variables into Equation (7).

## 2. Compute organizational choice.

Using the shadow wage approach implies market clearing even with minimum wages. Thus, labor supply is equal to labor demand at the shadow wage $n_{i j o}^{d,(k)}=n_{i j o}^{s,(k)}$. Then, we use wages $\left\{w_{i j o}^{(k)}\right\}_{\forall i j o}$ and labor demands $\left\{n_{i j o}^{d,(k)}\right\}_{\forall i j o}$ to compute the optimal organizational choice $\ell^{(k)}$ for all firms using the same method that we describe in the previous subsection. Note that wages and employment change when we update the firm's optimal organizational structure, e.g., we set the wage and employment of managers to zero for single-layer firms: $w_{i j m}^{(k)}=0$ and $n_{i j m}^{s,(k)}=n_{i j m}^{d,(k)}=0$ if $\ell^{(k)}=1$.

## 3. Compute markdowns.

For each occupation, we use the optimal organization $\ell^{(k)}$ and labor demand $n_{i j o}^{d,(k)}$ of each firm to compute its marginal revenue product of labor $\operatorname{mrpl}_{i j o}^{(k)}=\left.\frac{\partial y\left(z, \ell^{(k)}\right)}{\partial n_{i j o}}\right|_{n_{i j o}^{d,(k)}}$. Then, we use this marginal product $\operatorname{mrpl}_{i j o}^{(k)}$ and initial wages $w_{i j o}^{(k)}$ to compute:
(a) Minimum wage is not binding: Whenever the firm's wage and marginal product are both above the minimum wage, we compute the payroll market share $s_{i j o}^{(k)}$ using wages $w_{i j o}^{(k)}$ and labor demand $n_{i j o}^{d,(k)}$ of all firms in market $j$. Then, we compute the firm's structural elasticity $\varepsilon_{i j o}^{(k)}$ and markdown $\mu_{i j o}^{(k)}$ from Equations (12)-(13).
(b) Minimum wage is binding, and the labor supply equals the labor demand at the minimum wage: Whenever the minimum wage is binding and the firm's marginal product is above the minimum wage, we compute the firm's markdown from Equation (15).
(c) Minimum wage is binding, and the labor supply exceeds the labor demand at the minimum wage: Whenever the minimum wage is binding and the firm's marginal product is lower than or equal to the minimum wage, we set $\mu_{i j o}^{(k)}=1$.

## 4. Compute wages.

For all firms, we update the occupation-specific wages as follows:

$$
w_{i j o}^{\prime(k)}= \begin{cases}\max \left\{\underline{w}, \mu_{i j o}^{(k)} \cdot \operatorname{mrpl}_{i j o}^{(k)}\right\}, & \text { if } n_{i j o}^{(k)}>0 \\ 0, & \text { if } n_{i j o}^{(k)}=0\end{cases}
$$

We use these updated wages to construct $\mathbf{w}_{j o}^{\prime(k)}$ and $\mathbf{W}_{o}^{\prime(k)}$ using Equation (8).

## 5. Compute labor demand implied by minimum wages.

Here, we guarantee that the labor demand of constrained firms that face excess labor supply at the minimum wage is given by the inverse labor demand evaluated at the minimum wage. Particularly, we use the marginal product of labor $\mathrm{mrpl}_{i j o}^{(k)}$ and updated wages $w_{i j o}^{\prime(k)}$ to compute:
(a) Minimum wage is not binding: Whenever the firm's marginal product and wage are both above the minimum wage, the firm's labor demand coincides with the firm's labor supply $n_{i j o}^{\prime d,(k)}=n_{i j o}^{s,(k)}$.
(b) Minimum wage is binding, and the labor supply equals the labor demand at the minimum wage: Whenever the minimum wage is binding and the firm's marginal product is above the minimum wage, the firm's labor demand coincides with the firm's labor supply $n_{i j o}^{\prime d,(k)}=n_{i j o}^{s,(k)}$.
(c) Minimum wage is binding, and the labor supply exceeds the labor demand at the minimum wage: Whenever the minimum wage is binding and the firm's marginal product is lower than or equal to the minimum wage, we construct the labor demand of the firm from the employment level for which the minimum wage equals the marginal product of labor:

$$
\underline{w}=\left.\frac{\partial y\left(z, \ell^{(k)}\right)}{\partial n_{i j o}}\right|_{n_{i j o}^{\prime d,(k)}} .
$$

## 6. Update shadow wages.

The shadow wage is the wage that implies market clearing for all firms. That is, it does not coincide with the actual wage only for firms that face an excess of labor supply when they pay the minimum wage. We first use the updated employment levels $n_{i j o}^{\prime d,(k)}$ to update the market and aggregate employment levels from their definition:

$$
\begin{aligned}
\mathbf{n}_{j o}^{\prime d,(k)} & :=\left[\sum_{i=1}^{M_{j}}\left(n_{i j o}^{\prime d,(k)}\right)^{\frac{\eta_{o}+1}{\eta_{o}}}\right]^{\frac{\eta_{o}}{\eta_{o}+1}}, \\
\mathbf{N}_{o}^{\prime d,(k)} & :=\left[\int_{0}^{1}\left(\frac{\mathbf{n}_{j o}^{\prime} d,(k)}{B_{j o}}\right)^{\frac{\theta_{o}+1}{\theta_{o}}} d j\right]^{\frac{\theta_{o}}{\theta_{o}+1}}
\end{aligned} .
$$

Then, we update shadow wages:

$$
\tilde{w}_{i j o}^{(k+1)}=\left(\frac{1}{B_{j o}}\right)^{\frac{1+\theta_{o}}{\theta_{o}}}\left(\frac{n_{i j o}^{\prime d,(k)}}{\mathbf{n}_{j o}^{\prime d,(k)}}\right)^{\frac{1}{\eta_{o}}}\left(\frac{\mathbf{n}_{j o}^{\prime d,(k)}}{\mathbf{N}_{o}^{\prime d,(k)}}\right)^{\frac{1}{\theta_{o}}} \mathbf{W}_{o}^{\prime(k)}
$$

## 7. Iteration

Iterate over (1) to (6) until convergence of wages. Whenever max $\left\{\operatorname{abs}\left(w_{i j o}^{(k)}-w_{i j o}^{\prime(k)}\right)\right\}>$ tol, we update wages with the following criterion:

Unconstrained firms: $\quad w_{i j o}^{(k+1)}=\rho w_{i j o}^{(k)}+(1-\rho) w_{i j o}^{\prime(k)} \quad$ for $\quad \rho \in(0,1)$,
Any constrained firm: $\quad w_{i j o}^{(k+1)}=\underline{w}$.

## E Appendix: Quantification of the Model

## E. 1 Targeted Moments

Table E. 1 reports additional details to the quantification of the model parameters. In particular, it shows the model fit of each parameter to its most associated moment in the SMM estimation. The estimation brings about a close fit to the data, as it obtains an average absolute deviation of 5 percent between each model and data moment. The largest deviations occur in the span of control ( $7 \%$ ) and wage gap parameters ( $8 \%$ ).

## E. 2 Firm Substitutability Parameters

This section explains in detail the quantification of the parameters determining the structural labor supply elasticities: $\left(\eta_{o}, \theta_{o}\right)$.

## Across-market elasticity

We use municipality-level data on wages and employment between 2002 and 2016 to estimate the across-market labor supply elasticity. The specification takes the following form:

$$
\begin{equation*}
\log w_{m, o, t}=\beta \log L_{m, o, t}+\alpha_{m, o}+e_{m, o, t} \tag{C.26}
\end{equation*}
$$

where $w_{m, o, t}$ is the average wage in municipality $m, L_{m, o, t}$ is the total employment in municipality $m$, and $\alpha_{m, o}$ are municipality fixed effects for each occupation. Because employment

Table E.1: Targeted Moments

| Parameter Value | Description | Moment | Model | Data |
| :---: | :---: | :---: | :---: | :---: |
| A: Preferences |  |  |  |  |
| $\phi_{m}$ | Labor disutility shifter: production workers | Average firm size | 5.59 | 5.28 |
| $\phi_{w}$ | Labor disutility shifter: managers | Ratio managers to workers | 0.20 | 0.19 |
| B: Firm Organization |  |  |  |  |
| $\alpha$ | Span of control | Median span of control | 3.41 | 3.14 |
| $\varphi_{w}$ | Worker efficiency | Mean wage of workers ( $€$ ) | 729 | 718 |
| $\varphi_{m}$ | Managerial efficiency | Wage gap managers and workers | 0.79 | 0.73 |
| $\sigma_{z}$ | Std. Dev. firm TFP | Weighted mean HHI workers | 0.18 | 0.19 |
| C: Market Characteristics |  |  |  |  |
| $B_{i j w}$ | Amenities in small markets | Share workers in markets $M_{j} \leq 10$ | 0.12 | 0.12 |
| Mass $m_{j}=1$ | Share single-firm markets | Mass single-firm markets | 0.29 | 0.29 |
| $\zeta_{0}$ | Scale Pareto distribution | Mean № firms | 17.87 | 17.63 |
| $\zeta_{1}$ | Shape Pareto distribution | Std. Dev. № firms | 72.65 | 68.25 |
| D: Firm Substitutability |  |  |  |  |
| $\left(\theta_{w}, \theta_{m}\right)$ | Across-market firm substitutability | Across-municipality LS elasticity | $(1.52,0.92)$ | $(1.46,0.92)$ |
| $\left(\eta_{w}, \eta_{m}\right)$ | Within-market firm substitutability | Within-market LS elasticity | (19.97, 6.10) | (19.97, 6.10) |

Note: The Table reports the vector of parameters estimated using the SMM approach and the calibrated firm distribution with their respective moment description and fit.
and wages are jointly determined in equilibrium, we use the following shift-share instrument for $L_{m, o, t}$ :

$$
\begin{equation*}
\hat{L}_{m, o, t}=\sum_{s}(\underbrace{\frac{L_{i, m, s, o, 2002}}{\sum_{i} L_{i, m, s, o, 2002}}}_{\text {Industry-Municipality Share }} \times \underbrace{\sum_{i} L_{i, s, o, t}}_{\text {National Employment in Sector } s}) . \tag{C.27}
\end{equation*}
$$

The intuition for the instrument exploits across-municipality variation over time that stems from national employment shocks to sectors. The importance of each sectorial shift across municipalities depends on the sector's share in such municipality. Thus, municipalities vary in terms of exposure to the shift in sectors' employment. To explain why the instrument can be valid, we argue that multiple shifts to employment by sector, at the national level, are unrelated to local economic conditions. Hence, the national employment trends by sector exogenously adjust the local labor demand in this setup.

Regarding results, Table E. 2 shows the estimates from the IV regression of Equation (C.26). We find that the elasticity is lower for managers than for production workers. This suggests that it is more costly for managers to move across markets than for production workers.

Table E.2: Estimating the Across-Market Firm Substitutability Parameters

|  | $(1)$ <br> Production Workers | $(2)$ <br> Managers |
| :--- | :---: | :---: |
| Log employment | $0.433^{* * *}$ | $1.008^{* * *}$ |
| Municipality FE | $(0.027)$ | $(0.078)$ |
| Observations | Yes | Yes |
| Implied Elasticity $(1 / \beta)$ | 1,946 | 1,946 |
| Inferred across-market substitutability $\left(\theta_{o}\right)$ | 2.31 | 0.99 |

Note: The Table reports the estimates of the IV regression of Equation (17). Confidence intervals at the $95 \%$ level. The baseline period of the instrument is 2002, but we run the regression between 2009 and 2016 to exploit variation from the Great Recession. Standard errors are clustered at the municipality level. Source: 2002-2016, QP.

## Within-market elasticity

To estimate the within-market labor supply elasticity, we use establishment-level information on wages and employment between 2002 and 2016. We compute total employment $L_{i, j, o, t}$ and average hourly real wages $w_{i, j, o, t}$ in occupation o for each establishment $i$ in local labor market $j$ at period $t$. We do not need to impute working hours because our database already provides this information. For each occupation $o$, we separately estimate the following regression:

$$
\begin{equation*}
\log w_{i, j, o, t}=\beta \log L_{i, j, o, t}+\mu_{j, o, t}+v_{i, j, o, t}, \tag{C.28}
\end{equation*}
$$

We include local labor market-time fixed effects to isolate any time-varying shock in a given local labor market. Our goal is to estimate $\beta$, which represents the inverse of the withinmarket labor supply elasticity. The main threat to identification involves that the error $v_{i, j, o, t}$ captures establishment-time-specific shocks to labor demand and supply that are correlated
with establishment size. In that case, the OLS estimate of $\beta$ is biased. To address this problem, we additionally use a standard shift-share approach to simulate labor demand shocks that go back to Bartik (1991) and are formalized more recently through the exogeneity of the shares (Goldsmith-Pinkham et al., 2020) or the exogeneity of the shifts (Borusyak et al., 2022). The intuition of the instrument is that we exploit national trends in employment to predict establishment-level labor demand shocks. More concretely, we combine local shares and aggregate shifts to employment as follows:

$$
\begin{equation*}
\hat{L}_{i, j, o, t}=\underbrace{\frac{L_{i, j, o, 2002}}{\sum_{i} L_{i, s, o, 2002}}}_{\text {Firm's Employment Share in Sector } s} \times \underbrace{\sum_{i} L_{i, s, o, t}}_{\text {National Employment in Sector } s} \tag{C.29}
\end{equation*}
$$

We set 2002 as the initial year for the share component. Then, we measure the shares as the employment in occupation $o$ in a given establishment located in $j$ at $t\left(L_{i, j, 2002, o}\right)$ over the national level of employment for that occupation in sector $s$ in $2002\left(L_{i, j, 2002, o}\right)$. Recall that our definition of local labor market $j$ implicitly includes a sector, as it is the intersection between sector $(s)$ and municipality $(r)$ given an occupation $o$. We multiply this by the total employment of a given sector and occupation ( $\sum_{i}^{I} L_{i s o}$ ) every year after 2002 to predict current establishment employment according to national trends and initial shares. With the modified employment, we estimate Equation (C.28) by an instrumental variable with $\hat{L}_{i, j, o, t}$ as an instrument for $L_{i, j, o, t}$. In this estimation, we assume that the instrument is unrelated to unobserved constant or time-varying characteristics that affect specific establishments within the same industry, local labor market, and year.

Table E. 3 shows the IV estimates of the reduced-form elasticities by occupations. We find that managers have a smaller labor supply elasticity than production workers, indicating that managers are less responsive to wage differentials across firms within the same market. Overall, our estimates are in the range of the literature. Most estimates based on inverse methods, i.e., estimating the inverse labor supply elasticity in the baseline specification, find estimates around 5.24 (Sokolova and Sorensen, 2021). Quantifying the model to countylevel data in the U.S., Monte et al. (2018) finds a labor supply elasticity of 3.3. Using municipality-level German data, Ahlfeldt et al. (2015) find a labor supply elasticity of 5.5.

Table E.3: Estimating the Within-Market Firm Substitutability Parameters

|  | Production Workers |  | Managers |  |
| :--- | :---: | :---: | :---: | :---: |
| Dependent Variable: Log Wage | OLS | IV | OLS | IV |
| Log employment | $0.0500^{* * *}$ | $0.0623^{* * *}$ | $0.1639^{* * *}$ | $0.1645^{* * *}$ |
|  | $(0.0002)$ | $(0.0005)$ | $(0.0005)$ | $(0.0017)$ |
| Market-Year FE | Yes | Yes | Yes | Yes |
| Observations | $2,580,623$ | 459,960 | $1,036,216$ | 119,165 |
| Implied Elasticity $(1 / \beta)$ | 19.96 | 16.05 | 6.10 | 6.08 |
| Inferred within-market substitutability $\left(\eta_{o}\right)$ | 19.96 | 16.05 | 6.10 | 6.08 |

Note: The Table reports the estimates from regressing equation (19) by OLS and IV for each occupation. Standard errors in parentheses. Source: 2002-2016, QP.

## E. 3 Mass Layoff Shocks

This section explains in detail the definition and estimation of the effect of mass-layoff shocks on the municipality's employment and wages in the data. Then, we explain how we implement the same exercise in the model.

## Definition

To identify mass layoffs in the Portuguese data, we consider sudden, sizable, and enduring reductions in the employment size of a prominent establishment within the regional economy. More precisely, we define that a region suffers a mass-layoff shock when it experiences a drop of 100 workers in any establishment for two consecutive years between 2004 and 2016. ${ }^{19}$ All mass layoff events are aggregated at the municipal level to construct the treatment. So, we define the treated municipalities from the first time they experience a mass layoff in our sample period and those who never have one belong to the control group.

[^17]
## Estimation

To quantify the impact on local employment and wages, we use an event-study specification that compares the changes in employment and wages of treated and control municipalities for managers and production workers following a mass layoff. Our specification takes the following form:

$$
\begin{equation*}
y_{m t}=\xi+\sum_{k=-5}^{5} \beta_{k} \mathbf{1}\{k=t-g\}+\gamma_{m}+\gamma_{t}+X_{i t}^{\prime} \theta+\epsilon_{m t} . \tag{C.30}
\end{equation*}
$$

Here, the year that a mass layoff occurs in the establishment of a municipality is $g$, the years are $t$, and the event time indicators are $k .{ }^{20}$ Year fixed effects $\left(\gamma_{t}\right)$ and municipality fixed effects $\left(\gamma_{m}\right)$ control for unobserved constant characteristics across all municipalities and within municipalities, respectively. The covariates $X_{i t}$ control for baseline characteristics regarding the size and structure of the cities interacted with time to flexibly control for time-varying variables. More precisely, these variables are the $\log$ of the municipality's employment, the share of manufacturing employment, the share of highly educated workers, the share of male workers, and the share of young workers. The parameters of interest are $\beta_{k}$, which come from $k$ event time dummy variables. These dynamic treatment effects measure the effect on $y$ relative to an omitted period, which is when $k=-1$. We use Callaway and Sant'Anna (2021) estimator to control further for heterogeneous treatment effects across cohorts and aggregate all results across cohorts for the main coefficient shown in the main text. The main assumption needed in this setup is the conditional parallel trends assumption, stating that in the absence of mass layoffs, treated and control groups would evolve similarly once we net out baseline characteristics. Figure E. 1 shows that pre-treatment coefficients are not statistically different from zero, suggesting that this assumption holds. In addition, Figure E.1a shows the dynamic post-treatment coefficient on employment and wages following a mass layoff shock. Overall, the most negative significant results happen on the employment margin, not on the wage margin, for both types of workers, similar to the findings of Gathmann et al. (2020) in Germany.

[^18]Figure E.1: Event Study Estimates by Occupation


Note: These coefficients plot the estimated event study estimates using Callaway and Sant'Anna (2021) estimator. We exploit 72 events of mass layoffs across time in our sample period. We identify mass layoffs as a drop of 100 workers in a given establishment for two consecutive years. Source: QP, 2004-2016.

## Implementation in the model

We randomly group markets into municipalities and simulate two periods. The only difference between both periods is that the second contains a productivity shock randomly distributed across municipalities to firms that fulfill three characteristics that are informative of the firm's importance within the municipality and the size of the shock. First, we restrict the shock to multi-layer firms, as more than 95 percent of plants experiencing mass layoffs hire managers and production workers. Second, we exploit that the average firm carrying out a mass layoff had an average municipality employment share of 4.4 percent one year before the mass layoff. Thus, we condition the random shock on the sub-sample of firms whose municipality employment share falls within a specific range to match the same average size of shocked firms. Third, we choose the magnitude of the productivity shock to target that the average firm undergoing a mass layoff reduces its workforce by nearly 50 percent. Lastly, we run an OLS regression of employment and wages on a layoff dummy variable, controlling for municipality and time fixed effects.


[^0]:    *We are grateful to Felix Wellschmied and Jan Stuhler for their guidance and support, and to Antonia Díaz, Juan J. Dolado, Andrés Erosa, Luisa Fuster, Carlo Galli, Matthias Kredler, and Emircan Yurdagul for many insightful discussions. We are thankful for comments from Manuel Amador, Kyle Herkenhoff, Timothy J. Kehoe, and Simon Mongey, as well as all the participants at the Ph.D. Workshop at Universidad Carlos III and the Trade Workshop at the University of Minnesota. We also thank João Pereira dos Santos and Marta Lopes for their invaluable support to access the data. Álvaro Jáñez gratefully acknowledges support from the Ministerio de Ciencia e Innovación through research grants PRE2019088620, the AEI (Spain) through research grant PID2019-110779GB-I00, and Universidad Carlos III through the scholarship for Master's study. Lukas Delgado-Prieto acknowledges the financial support from the Ministry of Science and Innovation in Spain through research grant PDI2019$108144 \mathrm{~GB}-\mathrm{I} 00$.
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[^1]:    ${ }^{1}$ For evidence on monopsony power, see Staiger et al. (2010); Kline et al. (2019); Azar et al. (2022); Lamadon et al. (2022); Yeh et al. (2022). For evidence of its welfare effects, see Berger et al. (2022).
    ${ }^{2}$ We exclude CEOs and most managers are supervisors, team leaders, or middle managers. We define an occupational local labor market as the combination of a municipality and a 2-digit industry.

[^2]:    ${ }^{3}$ Occupation-based minimum wages are implemented in Australia, with its Modern Awards legislation, and are common in many European countries, whose collective contracts set occupation-specific wage floors.

[^3]:    ${ }^{4}$ According to occupation transitions, these broad categories represent a persistent occupational state. Figure A. 5 shows that most workers remain within the same category after changing to another establishment.
    ${ }^{5}$ This measure is broadly used in the literature (Azar et al., 2020; Berger et al., 2022; Azkarate-Askasua and Zerecero, 2023; Jarosch et al., 2023), and it is the baseline measure of the U.S. Department of Justice to evaluate the competitive effect of mergers.

[^4]:    ${ }^{6}$ This observation is robust to measuring HHI with the employment share rather than the payroll share.

[^5]:    ${ }^{7}$ We do not weigh each market by employment size to isolate the decomposition from employment sorting.

[^6]:    ${ }^{8}$ In Figure A.4, we also show that this positive relationship holds when we take the unweighted average of the share of multi-layer firms across local labor markets.

[^7]:    ${ }^{9}$ In the model, the municipality is a collection of local labor markets. We use a municipality-level regression, instead of a market-level regression, to use the standard formulation of the shift-share instrument.

[^8]:    ${ }^{10}$ Comparing mobility rates, nearly 10 percent of workers change municipality in Portugal yearly. Instead, the yearly migration rate across U.S. counties, subsets of commuting zones, ranges between 3 to 6 percent in the same period (Molloy et al., 2011).

[^9]:    ${ }^{11}$ These papers find that geographic mobility accounts for most of the regional employment change. For instance, Gathmann et al. (2020) finds that outflows to non-employment account for only 20 percent of the overall employment decline in German regions following a mass layoff. They also find that geographic mobility protects workers below 50 from suffering employment losses.

[^10]:    ${ }^{12}$ In nominal terms, the Portuguese minimum wage increased from $530 €$ to $600 €$. Adjusting for the CPI, where we set the base year to 2010 , it implies an increase of 10 percent from $525 €$ to $578 €$. For more information, see: https://www.dgert.gov.pt/evolucao-da-remuneracao-minima-mensal-garantida-rmmg.

[^11]:    ${ }^{13} \mathrm{We}$ estimate the elasticity as $O W E_{o}=\frac{\% \Delta \text { Employment }_{o}}{\% \Delta \text { MinimumWage }} * \frac{\% \Delta \text { MeanWage }_{o}}{\% \Delta \text { MinimumWage }^{-1}}$.

[^12]:    ${ }^{14}$ Under the Modern Awards system, the Australian government implements statutory minimum wages that are occupation-based. Moreover, collective agreements that set distinct wage floors across occupations in an industry are common in European countries such as Italy (Adamopoulou et al., 2023), France (Fougère et al., 2018), Belgium, Sweden (International Labour Organization, 2023), and Portugal. However, Portuguese workers typically earn significant wage premiums over their wage floor (Card and Cardoso, 2022), indicating that wage bargaining is not effective in preventing firm-wage setting power for most employees.

[^13]:    ${ }^{15}$ See Table A. 1 for further information about the categories of the occupational classification, which is based on Decreto-Lei $n^{o}$. 121/78 de 2 de Junho, Ministério do Trabalho.

[^14]:    ${ }^{16}$ For a comprehensive guide to additive random utility and nested models, see Chapter 15 in Cameron and Trivedi (2005).

[^15]:    ${ }^{17}$ For simplicity, we omit the non-negativity constraints associated with consumption and labor supply.
    In the unconstrained solution, we will observe that such values also satisfy the constrained solution because they are always greater than or equal to zero.

[^16]:    ${ }^{18}$ Furthermore, we note that the algorithm does not improve in terms of correctly solving Problem (9) when we apply more times.

[^17]:    ${ }^{19}$ We further restrict establishments with more than 1 percent of local employment in the baseline period, and we do not take into account plant closures to define mass layoffs.

[^18]:    ${ }^{20}$ We define the cohorts of treated municipalities from the first time, in our sample period, they had a mass layoff. Other mass layoffs may happen in the same municipality later on.

